AD 748621

MEASUREMENTS

OF

MILITARY ESSENTIALITY

LMI Task 72-3

August 1972

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13. ABSTRACT One of the major unsolved military logistics problems of the past twenty years has been the search for an objective, rational method to consider military essentiality in the preparation of budgets, procurement plans and allowance lists for parts and components. Underlying each stock control policy (e.g., reorder point, safety level, reorder quantity) is an assumed or implied value of the military essentiality or worth of the part or component to which it applies. However, in most instances objective measurements of military essentiality or worth are not made. Instead, largely arbitrary estimates are substituted; e.g., a cost is assigned to a stock-out. While a great deal of effort and talent has been spent on the development and refinement of sophisticated inventory models, uncertainty resulting from an absence of objective measures of military essentiality has tended to nullify the improvements which were implied by such refinements.

This report describes a method which will allow military essentiality to be weighed objectively in budgets and procurement plans for repairable (recoverable) components. Using the method proposed, spares would be procured so that the number or percentage of operational units of the different weapon systems (e.g., B-52s, F-4s) would be in the best balance, in the opinion of high level planners, for any or all funding constraints.

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FOREWORD

Although this report fulfills the requirements of Task 72-3 it should be considered as a mid-project interim report. The model described will be refined and tested under a new Air Force task. This report is primarily a working document for those individuals involved in the forthcoming tests. At the conclusion of the tests a report will be prepared for general distribution. That later report will describe a generalized model and will discuss the applicability and benefits of that model.

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I. INTRODUCTION

Under Task 72-3 (Appendix 1) entitled "Measurements[1] of Military Essentiality" LMI was asked to:

- Review methods which have been developed for considering the military essentiality of parts and components in the development of procurement and distribution plans
- Evaluate the need and feasibility of developing additional methods for use by the Air Force
- Recommend methods to be used by the Air Force.

A literature search revealed seventy-five papers which address the problem of considering or measuring military essentiality. The more relevant of those are enumerated in Appendix 2. Findings resulting from analysis of available literature are summarized below.

1. We were unable to locate an existing model which provided a reasonable way to consider the military essentiality of parts or components:
   - For the development of procurement plans

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[1]As will be seen in later sections, the objective of "measuring" the military essentiality of parts and components, although technically valid, can be misleading. The principal model proposed in this report can be interpreted as one which eliminates the need to "measure" military essentiality by substituting a methodology which "guarantees" that procurement plans are consistent with high level military judgments regarding the balancing of forces and the relative importance of elements of those forces.

- When more than one aircraft type[1] is to be supported by the same fund source.

Few of the models reviewed addressed the above problem. Those that did assumed that aircraft types could be ranked (or assigned values) in terms of military importance: e.g., B-52s, as a class, are more important than F-4s, as a class; or, B-52s, as a class, are twice as important as F-4s, as a class. This assumption ignores the fact that forces should be balanced as to quantities of different types of weapon systems and that the optimum balance in terms of relative quantities may change as costs and available funds vary. Section II.D describes a model LMI developed to eliminate that and other weaknesses.

2. One material distribution model developed by the Air Force considers military essentiality adequately. Concepts incorporated in that model should be used in the development of distribution plans for recoverable (repairable) components. Section IV discusses that Air Force model and its applicability.

3. An Air Force model together with an Air Force codification system can be adapted and extended in such a way that the military essentiality can be considered objectively in the development of procurement plans and budgets for recoverable components. Section II explains pertinent aspects of that Air Force model (and

[1]Throughout this report the term weapon system type could be substituted for the term aircraft type without introducing conceptual errors. Aircraft type refers to the combination of a mission symbol and a design number. Thus F-4 aircraft comprise a type.

the associated codification system) and describes a military essentiality model developed by LMI which exploits them.

4. The Air Force and LMI models mentioned in 3 above can be adapted to the development of procurement plans and budgets for certain high cost consumable parts considering military essentiality. See Section III.

5. No models were found, and none was developed by LMI, which could be recommended for considering military essentiality in the development of procurement plans for low cost consumable parts. Although the model recommended in Section II could be adapted for this purpose it is believed that the costs would be prohibitive considering the potential benefits.

II. A MODEL TO CONSIDER MILITARY ESSENTIALITY IN THE DEVELOPMENT OF PROCUREMENT PLANS AND BUDGETS FOR RECOVERABLE COMPONENTS

A. PREREQUISITE SYSTEMS

Two Air Force systems already developed are exploited in this model. They are the Air Force Recoverable Item Classification System and the Air Force METRIC[1] System. These two systems are discussed briefly below.

1. Air Force Recoverable Item Classification System

Under this system recoverable components[2] were coded to indicate the effect that their failure, in the absence of a serviceable spare, would have upon the weapon systems they support. In response to an AFLC letter MCNRRC dated 19 February 1970, item managers in concert with equipment specialists evaluated more than 180,000 recoverable components and classified them into one of the following five categories.

Category	Impact
A	An item of supply whose failure, in the absence of a serviceable spare, prevents primary and secondary missions from being accomplished, weapons from operating, or presents a hazard to the safety of the occupants or users of the item or end item.

[1] METRIC is an acronym for "Multi-Echelon Technique for Recoverable Item Control." The METRIC System is part of the overall Air Force Advanced Logistics System (ALS).

[2] Throughout this report a given "component" is a set of interchangeable (substitutable) units. A "recoverable component" is one such that units of the set normally can be repaired and used again.

Category	Impact
B	An item of supply whose failure, in the absence of a serviceable spare, presents a not fully equipped (NFE) status in which the primary mission cannot be accomplished, however, secondary missions can be accomplished.
C	An item of supply whose failure, in the absence of a serviceable spare, presents a not fully equipped (NFE) status in which secondary missions cannot be accomplished and/or the primary mission is impaired.
D	An item of supply whose failure, in the absence of a serviceable spare, presents a not fully equipped (NFE) status, but does not materially affect primary or secondary mission accomplishment.
E	An item of supply not eligible for classifying in one of the other four categories (Codes A-D).

In brief, any component classified as "A" or "B" will cause an aircraft to be NORS (not operationally ready because of supply) when the component is not operational and a spare is not readily available. Components classified category "C" can cause an aircraft to be in a serious NFE (not fully equipped) status. Components classified "D" can cause an aircraft to be in a minor NFE status. Lack of a component classified "E" will have little or no consequence.

2. METRIC System

The METRIC System was originally proposed by the RAND Corporation in 1966.[1] AFLC has designed, programmed and tested a modified version of the METRIC System. Modifications and improvements are still being made. We will not describe how the current METRIC System operates. Instead we will discuss a

[1] Sherbrooke, C. C. (1966): METRIC: A Multi-Echelon Technique for Recoverable Item Control, The RAND Corporation, RM-5078-PR.

fundamental concept inherent in one of the METRIC subsystems, called the requirements subsystem. That subsystem incorporates a LaGrange multiplier method to derive a procurement plan for spare recoverable components. Under one of the available options, it produces a procurement plan[1] which is optimal in the following sense. For each component the quantity of spare units to be procured is specified. That quantity is such that the "expected back order reduction per unit cost" obtained from the last unit (and all earlier units) to be procured is equal to or greater than some specified value (called the "shadow price"). Furthermore, the expected back order reduction per unit cost that would be obtained from procuring one additional unit would be less than that shadow price. The details of how computations are made by this METRIC subsystem are not germane to this report. What is germane is the fact that the computation procedures used makes it possible to modify extant computer programs in such a way that the back order reductions expected from procurement of each spare <u>unit</u> (i.e., the first, the second, the third, etc.) of each component can be extracted from the program. In subsequent sections we will describe a program written by LMI to extract this information.

B. ORDER OF DEVELOPMENT

To clarify the purpose and interrelationships of the various elements of the recommended model, we will first describe how to consider military essentiality in a relatively simple hypothetical problem. We will then show how to build upon the solution to the

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[1]The subsystem calculates system-wide spare component stock levels. Those stock levels can be used to derive either procurement plans or budgets depending on the way constraints are imposed.

simple hypothetical problem in such a way that the actual Air Force problem can be solved. The order of development will be as follows:

- We will first discuss a model which would be applicable if all components on an aircraft were NORS-causing recoverables, and the Air Force only utilized one aircraft type.
- Next, we will describe how to extend the model, realizing that the Air Force utilizes many aircraft types.
- We will then discuss how to further extend the model, realizing that many components cannot cause airplanes to be NORS. In that section, we will also describe how to consider the fact that some NFE-causing components degrade the capabilities of an aircraft more than others.
- Lastly, we will discuss how the model can be modified to consider a multitude of factors ignored to that point. In particular we will describe ways to consider real life complications such as the following.
  - -- Some recoverable components are used on more than one aircraft type
  - -- Some NORS-causing items are not recoverable components
  - -- Cannibalization[1] can sometimes be used to reduce the effect of spare component shortages.

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[1]Cannibalization is the process whereby a good component is removed from an aircraft which is NORS for other reasons and is then used to make a different aircraft operational.

The text of this report, pages 8-37, will describe how the proposed model operates and how it should be used. Technical justification, mathematical formulations and proofs, and discussion of theoretical aspects are contained in Appendix 3.

C. A MODEL APPLICABLE TO NORS-CAUSING RECOVERABLE COMPONENTS CONSIDERING ONLY ONE AIRCRAFT TYPE

1. Perspective

Assume the following hypothetical conditions.

- The Air Force utilizes only one aircraft type
- All components on that aircraft type are NORS causing recoverable components
- A NORS condition can only be caused by such recoverable components

Under such circumstances, with a given funding constraint, a component procurement model which minimizes the expected number[1] of NORS aircraft gives proper consideration to military essentiality. In this section, we describe such a model--a model which employs the METRIC requirements subsystem.

2. Expected Marginal Back Order Reductions

LMI has designed a computer program which, when used in conjunction with the METRIC requirements subsystem discussed above (henceforth called METRIC) will compute the expected back order reduction for each spare unit (the 1st, 2nd, 3rd ...) of each recoverable component. The LMI program is simply inserted in the METRIC Program. Henceforth we will call the combined program the METRIC-LMI Program.

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[1]The expected number is the computed probability that a random aircraft, at a random point in time, will be NORS multiplied by the total number of aircraft.

The input data required for the METRIC-LMI Program are the same (and are in the same format) as those required for one option of the METRIC Program. For each component to be considered, input data required by the METRIC-LMI Program are as follows.

- Component name (or number)
- Unit Cost
- Number of bases which have demands for the component
- Expected quantity in the base repair pipeline. This quantity is the predicted average number of end use demands[1] per time period, multiplied by the percent of the component repaired at base, multiplied by the average number of time periods required to repair a unit of a component (of this type) at a base.
- Expected quantity in the depot repair pipeline. This is the predicted average number of end use demands per time period, multiplied by the percent repaired at the depot, multiplied by the average number of time periods required to ship units of the component to a depot and to repair it at the depot.
- Expected transportation pipeline. The predicted average number of end use demands per time period, multiplied by the percent repaired at the depot, multiplied by the average number of time periods required to ship units of the component to a base from the depot.

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[1]End use demands are those which are made upon a base by an aircraft. Demands made by a base on a depot are ignored since they cannot cause a NORS condition directly.

- Variance to mean ratio. This parameter indicates the spread or dispersion of the probability distribution used to predict demands. It is a measure of how much the demand rates for a component are expected to fluctuate.

The output of the METRIC-LMI Program indicates for each component the average number of back orders expected, given any quantity (0, 1, 2, ... etc.) of spares in the system. It also indicates for each component the expected back order reduction which will be obtained by each spare unit (the 1^{st}, 2^{nd}, 3^{rd}, ... etc.). The procedure used is similar to that suggested by Sherbrooke on pages 14-16 of RAND Memorandum RM 5078-PR.[1]

Figure 1 is a schematic representation of the METRIC-LMI Program and its output. Section B of Appendix 3 contains a brief description of the logic used in the program.

3. Expected NORS Reductions

Objective

With a given funding level the minimization of expected back orders (the same as maximization of expected back order reductions) across components does not necessarily[2] minimize the expected number of NORS aircraft. Thus a model was needed which converted expected back orders (and expected back order reductions)

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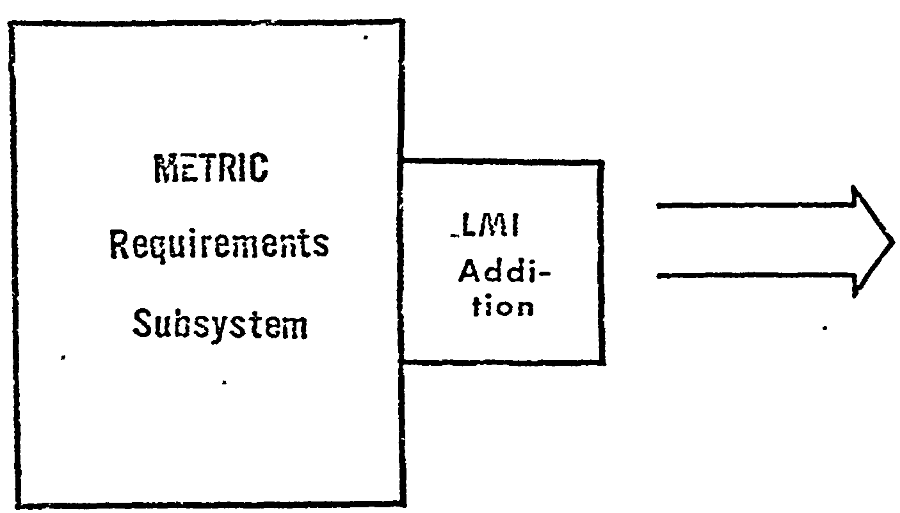
[1] Sherbrooke, METRIC: pp. 14-16.

[2] In some, and possibly most, real life problems the minimization of expected back orders provides a solution which is almost identical to one obtained by minimizing NORS aircraft. However, the difference could be significant. (See Appendix 3, Section D.)

SCHEMATIC OF METRIC-LMI PROGRAM AND OUTPUT

METRIC-LMI MODEL



Qty.	Item	Expected End Use Back Orders	Back Order Reduction
0	A	5.6701	
1	A	4.9263	.7438
2	A	4.3151	.6112
3	A	3.8150	.5001
		. . . .	
0	B	8.6041	
1	B	7.9111	.6930
2	B	7.4389	.4722
		. . . .	

Figure 1

into expected NORS aircraft (and expected NORS reductions), and guaranteed the minimization of expected NORS within available funds. It was also necessary to develop a model to compute the expected number of NORS aircraft, given the availability of spares of each component.[1] The LMI NORS model described below was developed and programmed to accomplish those objectives and to operate in conjunction with the METRIC-LMI Program. Figure 2 is a schematic representation of the METRIC-LMI and the LMI NORS models. That figure will be referenced as the models are explained.

Assumptions

The LMI NORS model assumes:

- An aircraft missing one or more NORS-causing recoverable component units because a spare is not available will be NORS.
- An aircraft cannot be NORS unless at least one unit of a NORS-causing component is in need of repair and a spare is not available.
- The failure (or need for repair) of a NORS-causing component unit is independent of the failure of other component units and is independent of the state (NORS or NOT NORS) of the aircraft on which it is installed.
- When more than one unit of a component are in stalled on an aircraft, the failure of any one is independent of the failure of any of the others.

[1]The need for this computation will become clear when multiple aircraft types are considered in Section II.D.

SCHEMATIC OF METRIC-LMI AND LMI NORS MODELS

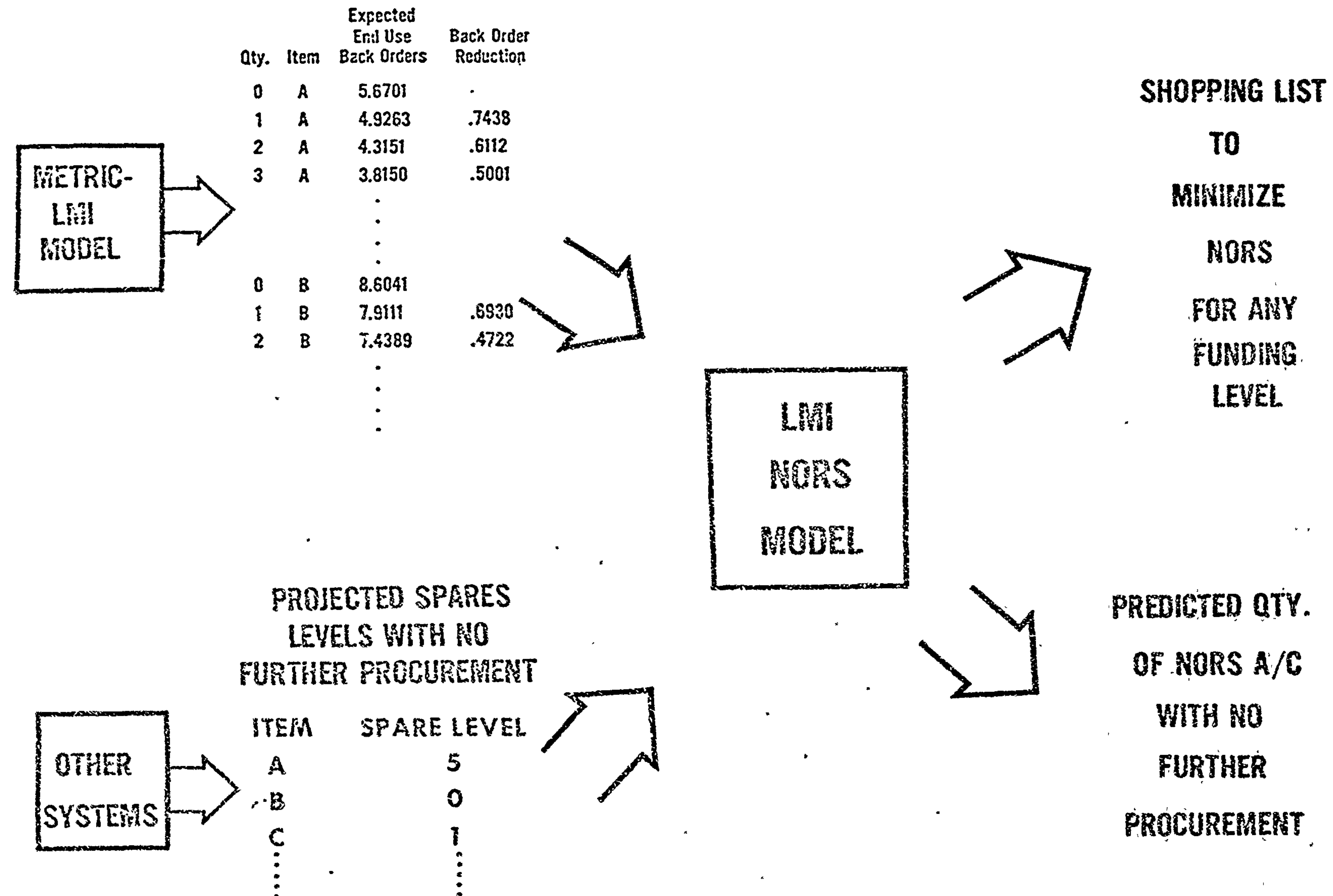


Figure 2

The above assumptions represent only a first approximation of the actual Air Force environment. Section II.G will describe modifications and enhancements (to the model now being described) designed to make the final calculations more consistent with the actual environment.

Output

The LMI NORS model (together with a computer program based on that model) produces two output documents which are required for military essentiality to be given correct emphasis in the development of procurement plans and budgets.

- Predicted NORS

For a given aircraft type the LMI NORS model computes the expected quantity of NORS aircraft for any given inventory of spare components. For instance, if one were given the inventory, at some future time, of each of the many recoverable components used on the aircraft being considered, the model could compute the quantity of those aircraft expected to be NORS at that time.

Figure 2, exclusive of the upper right quadrant, is a schematic diagram of the use of the LMI NORS model to predict the expected quantity of NORS aircraft.

- Optimum Procurement Plans and Budgets

The model also produces a single ordered list of spare units of components. From that list optimum procurement plans (and corresponding budgets) for any and all fund constraints which might be imposed can be derived. We call the list the "shopping list." Every unavailable (unprocured previously and not in the projected inventory) spare unit of every

component will have a unique position in the shopping list and it will be so ordered (sorted) that the first k units (involving many different components) found in the list will represent an optimum procurement plan for the amount of money they will cost. In other other words as one procures units in the order of the list he can be assured that at each step[1] he has reduced the expected number of NORS aircraft the maximum amount possible considering the amount of money spent to that point. Appendix 3 describes and justifies the procedure used for generating shopping lists.

Figure 3 depicts a hypothetical shopping list and illustrates an optimum procurement plan. Note in particular the columns which represent cumulative NORS reductions and cumulative cost. Those columns are referred to a number of times in the following section.

Figure 2, exclusive of the lower right quadrant, is a schematic diagram of the use of the LMI NORS model to produce shopping lists.

D. EXTENSION OF THE MODEL TO CONSIDER MULTIPLE AIRCRAFT TYPES

1. Perspective

Given the assumptions listed on page 12, and given an amount of money to spend for spare components, the model described

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[1]A legitimate step down the list might require the procurement of a set of spare units of the same component. For instance, when components, such as brakes, fail infrequently but usually fail in pairs, then a legitimate step down the ordered list might call for the procurement of the 1st and 2nd brake as a pair of brakes. The list developed by the LMI NORS model indicates all legitimate stopping points. In the example of the brakes, the list would indicate that the 1st brake does not constitute a legitimate stopping point while the 2nd brake does. See Appendix 3, Section D.

HYPOTHETICAL B-52 SHOPPING LIST

Unit & Component	NORS Reduction**	Cumulative NORS Reduction	Unit Cost	Cumulative Cost	Sort Values**
6^{th} A*	.03108	.03108	1,598	1,598	$.403172 \cdot 10^{-7}$
1^{st} B	.04147	.07255	2,300	3,898	$.400377 \cdot 10^{-7}$
2^{nd} C	.18749	.26004	10,400	14,298	$.400198 \cdot 10^{-7}$
2^{nd} B	.03532	.29536	2,300	16,598	$.377182 \cdot 10^{-7}$
1^{st} D	.20969	.50505	13,800	30,398	$.330428 \cdot 10^{-7}$
7^{th} A	.02333	.52838	1,598	31,996	$.316683 \cdot 10^{-7}$
2^{nd} E	.00276	.53114	202	32,198	$.310552 \cdot 10^{-7}$
⋮	⋮		⋮	⋮	⋮

The sort criterion guarantees that the procurement of 2 As, 2 Bs, 1 C and 1 D will maximize NORS reductions given $31,996.

* Without additional procurement five A spares will be available.

** For explanation see Appendix 3.

Figure 3

in II.C above minimizes NORS aircraft within a single aircraft type. It maximizes the number of operating aircraft and thus would maximize military readiness[1] if the Air Force utilized only one aircraft type. In fact, the Air Force utilizes many aircraft types. Thus a model is needed to obtain an optimum component procurement plan across various aircraft types. That model should provide the best balanced set of forces considering costs and funds available. The model discussed below was developed for this purpose. Note that the model does not utilize the concept of assigning a relative worth value to an aircraft type. Instead, it assumes that the importance of increasing by one the number of operational aircraft of a given type varies as costs, available funds, and the existing balance of forces vary. Complications which will be addressed include the following.

- As the NORS rate for a given aircraft type is reduced by augmenting spare component levels, the cost of an incremental reduction in the rate will usually increase.
- The cost of reducing the NORS rate by a fixed increment will vary from aircraft type to aircraft type.
- Since the Air Force operates with limited funds, the best balance of forces will depend upon relative <u>costs</u>. For example, if the unit cost of procuring A-7s was 25% of the F-4 unit cost, the ratio of A-7s to F-4s in the best obtainable force might be Y. If, on the other hand,

---

[1]Military readiness would be increased the maximum amount possible by obtainment of spare components with funds made available specifically for the obtainment of such spares.

the unit of cost of procuring A-7s was 100% of the F-4 unit cost, the ratio of A-7s to F-4s in the best obtainable force would be less than Y (but not necessarily Y/4).

The model discussed below considers the cost[1] and worth of each operational unit[2] of each aircraft type. The units are those which could be removed from NORS status and made operational by the procurement of additional spare components.

2. The Extended Model

The extension is basically a procedure which allows high level military planners to make decisions in such a way that the procurement plans for spare components will reflect those decisions directly.

Before planners are asked to make military decisions the LMI NORS model is run for every aircraft type to be considered. We will illustrate how the extension operates by assuming that the Air Force utilizes only three aircraft types, the F-4, B-52, and the KC-135.

The first step in the procedure is use of the LMI NORS model to predict the number of NORS and Not-NORS aircraft which can be expected at some future date if the current and projected

---

[1]Cost in this instance does not include the original acquisition cost. Instead the cost considered is that required to change the status of on hand aircraft from NORS to NOT NORS.

[2]An operational unit of an aircraft type will be defined by the following example. Suppose that at a given point in time the expected number of operational aircraft of type T is 600. Suppose that thereafter enough spare units are added to the system to increase the expected number of operational aircraft of type T to 601. At that time we have obtained the 601st operational unit of aircraft type T.

spares levels are not augmented with additional procurement. Figure 4 is a hypothetical projection produced by the LMI NORS model.

Next the LMI NORS model is used to develop shopping lists, of the type illustrated in Figure 3, for each aircraft type. Each list represents optimum procurement plans for all possible fund allocations to the associated aircraft type. Hypothetical lists for three aircraft types are illustrated in Figure 5. Each list contains those units, and only those units of spares which are not expected to be available unless additional procurement is undertaken. For example, the first unit of Component A that appears in the B-52 list (Figure 5) is the 6th unit. This means that it had been predicted that five Component A spares would be available in the system (as a whole) if no additional procurement is made. Note that the lists contain cumulative NORS and cumulative cost columns, and from those columns the cost of obtaining each additional operational unit of each aircraft type can be ascertained. For instance, in the illustration (Figure 5) the first cumulative NORS reduction of B-52s would provide the 701st operational B-52 at a cost of $70,000. The second NORS reduction would provide the 702nd B-52 at a cost of $120,000.

With these lists it now becomes possible to obtain military judgments relating the worth of the individual units of the various aircraft types to their costs. Notice that the first B-52 NORS reduction (which would provide the 701st operational B-52) would cost $70,000. Notice also that $70,000 could be used instead to reduce F-4 NORS by three (providing the 1201st, 1202nd, and 1203rd operational F-4s.) Likewise $70,000 could be used to reduce KC-135 NORS by two (providing the 851st and 852nd KC-135s).

HYPOTHETICAL PROJECTION PRODUCED BY LMI NORS MODEL

STARTING POSITION

(No Procurement of Additional Spares)

	TOTAL	NOT NORS	NORS
B52	800	700	100
F4	1500	1200	300
KC135	900	850	50

Figure 4

SHOPPING LISTS TO BE MERGED

(INITIAL POSITION)

B-52			
Unit & Component	Cumulative NORS Reduction	Cumulative Cost	
6^{th} A	.03108	1,598	
1^{st} B	.07255	3,898	
. . .	1	70,000	
. . .			Unit Cost 120,000
. . .	2	190,000	
. . .			Unit Cost 180,000
. . .	3	370,000	

F-4		
Unit & Component	Cumulative NORS Reduction	Cumulative Cost
. . .	3	70,000
. . .	5	120,000
. . .	6	180,000

KC-135		
Unit & Component	Cumulative NORS Reduction	Cumulative Cost
. . .	2	70,000
. . .	3	120,000
. . .	4	180,000

Figure 5

Having this information, it becomes possible to ask high level planners the first of a series of questions relating to the worth of units of aircraft types to costs. The first question in the hypothetical example would be:

> "With your first increment of money (available to procure spare components) which of the following would you prefer to obtain:
>
> a. One additional operating B-52 (the 701^{st});
>
> b. Three additional operating F-4s (the 1201^{st}, 1202^{nd}, and 1203^{rd}); or
>
> c. Two additional operating KC-135s (the 851^{st} and 852^{nd})?"

Assume that the selection made is option (a), i.e., one additional B-52. Two consequences of this decision are immediately obvious.

- Component units in the B-52 list up to the point where the cumulative NORS reduction equals one should compose the first increment of an overall multi-aircraft type shopping list (see Figure 6).

- Subsequent military judgments should be based on comparison of component units and aircraft units remaining, i.e., those not already selected (see Figure 6).

In this hypothetical case the second B-52 NORS reduction would cost \$120,000 (\$190,000 required to obtain the first two cumulative NORS reduction minus \$70,000 required to obtain the first). Therefore the next question would be

SITUATION AFTER FIRST DECISION

NEW STARTING POSITION

	NOT NORS	NORS
B-52	701	99
F-4	1200	300
KC-135	850	50

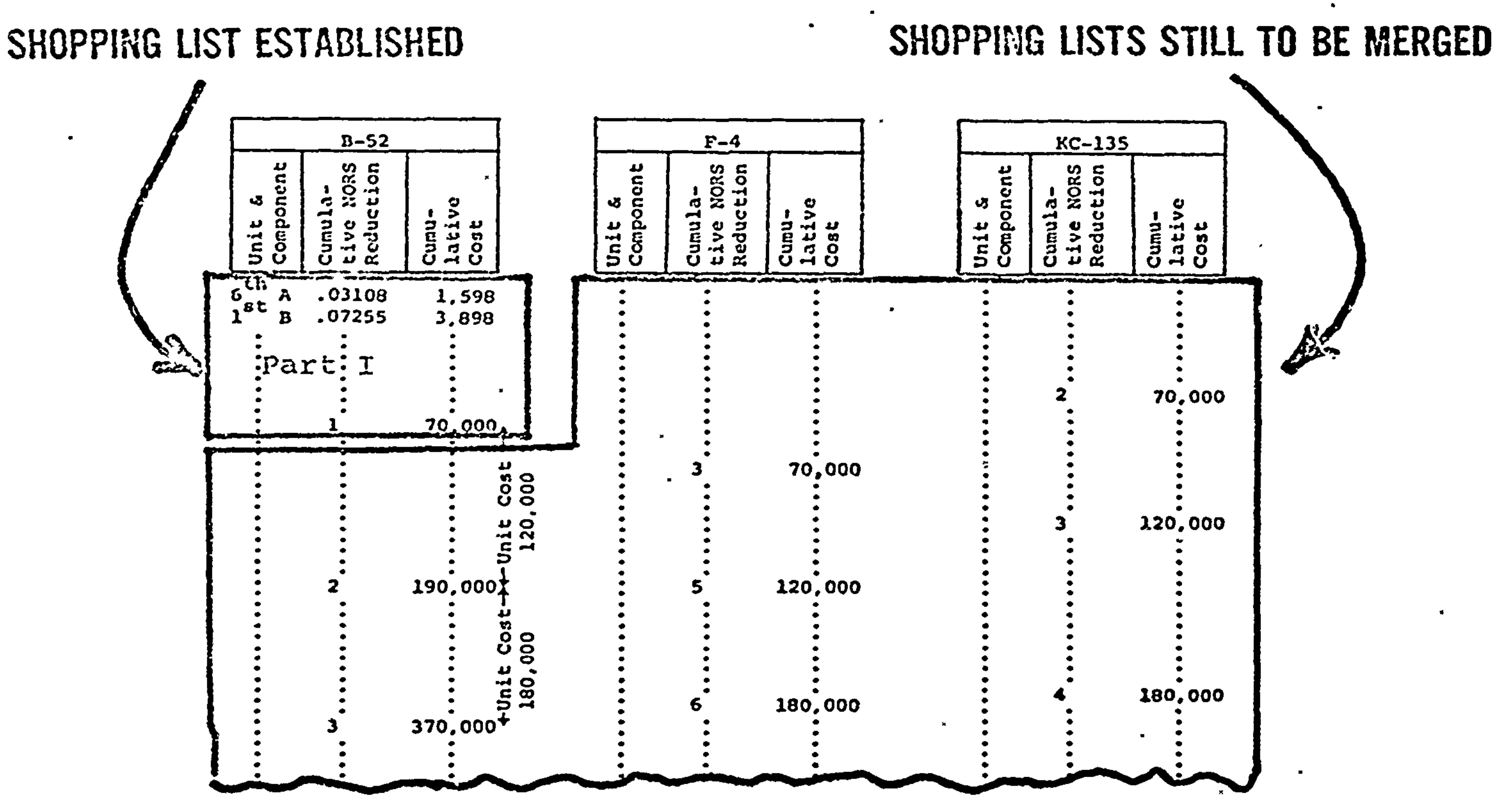

Figure 6

"With your next increment of money, which would you prefer:

a. One more B-52 (the 702nd);

b. Five F-4s (the 1201st through the 1205th), or

c. Three KC-135s (the 851st through the 853rd)?"

Assume again the preference is one B-52. Now the component units constituting the second increment of an integrated component shopping list are known and a third question can be formulated. (See Figure 7.) Since the next available B-52 NORS reduction (the third one) will cost $180,000, and since no F-4 nor KC-135 aircraft have yet been selected, the third set of options would be:

a. One more B-52 (the 703rd),

b. Six F-4s (the 1201st through the 1206th), or

c. Four KC-135s (the 851st through the 854th).

If the choice in this case is "six F-4s" the shopping list would be incremented as illustrated by Figure 8 and the next choice might be a) one B-52 (the 703rd), b) three[1] F-4s (the 1207th through the 1209th), or c) four KC-135s (the 851st through the 854th).

The process described above is continued until the unified shopping list contains enough entries to constitute an optimum procurement list for any funding allocation.

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[1]The low cost F-4s have now been selected. Therefore the cost per F-4 has been increased and the relative quantity corresponding to one B-52 or four KC-135s has been decreased.

SITUATION AFTER SECOND DECISION

NEW STARTING POSITION

	NOT NORS	NORS
B-52	702	98
F-4	1200	300
KC-135	850	50

SHOPPING LIST ESTABLISHED

SHOPPING LISTS STILL TO BE MERGED

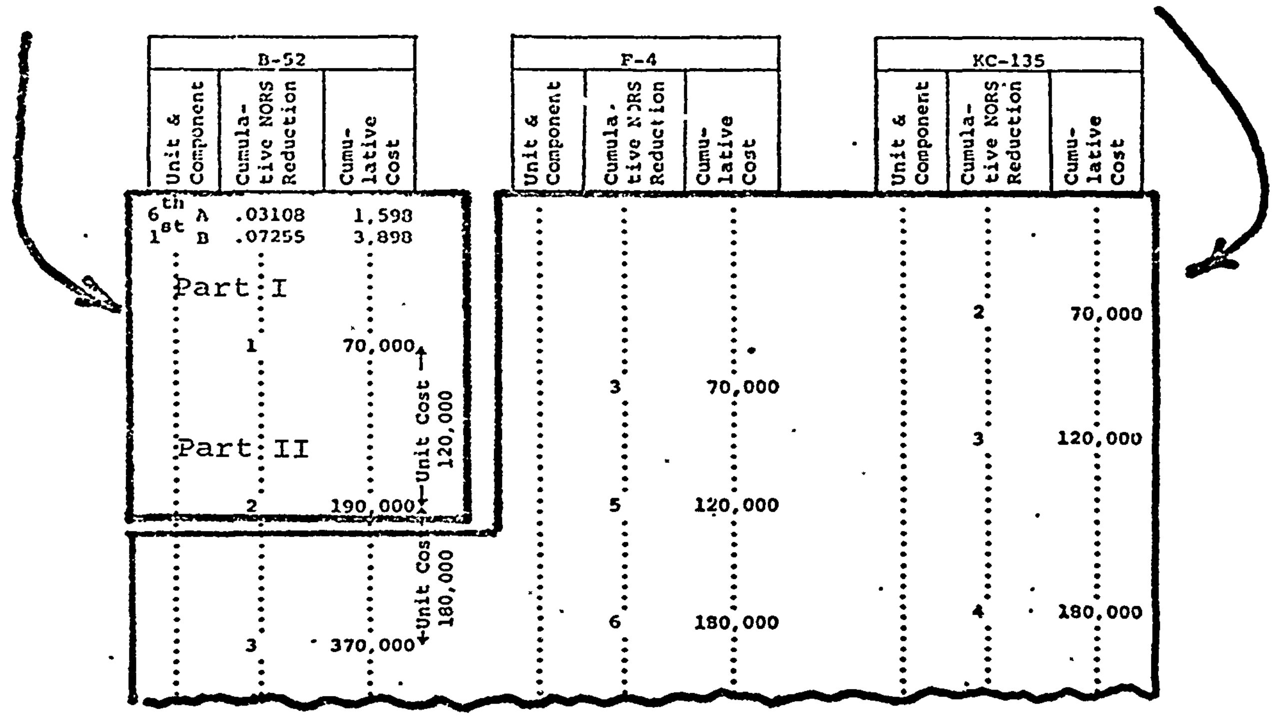

Figure 7

SITUATION AFTER THIRD DECISION

NEW STARTING POSITION

	NOT NORS	NORS
B-52	702	98
F-4	1206	294
KC-135	850	50

SHOPPING LIST ESTABLISHED

SHOPPING LISTS STILL TO BE MERGED

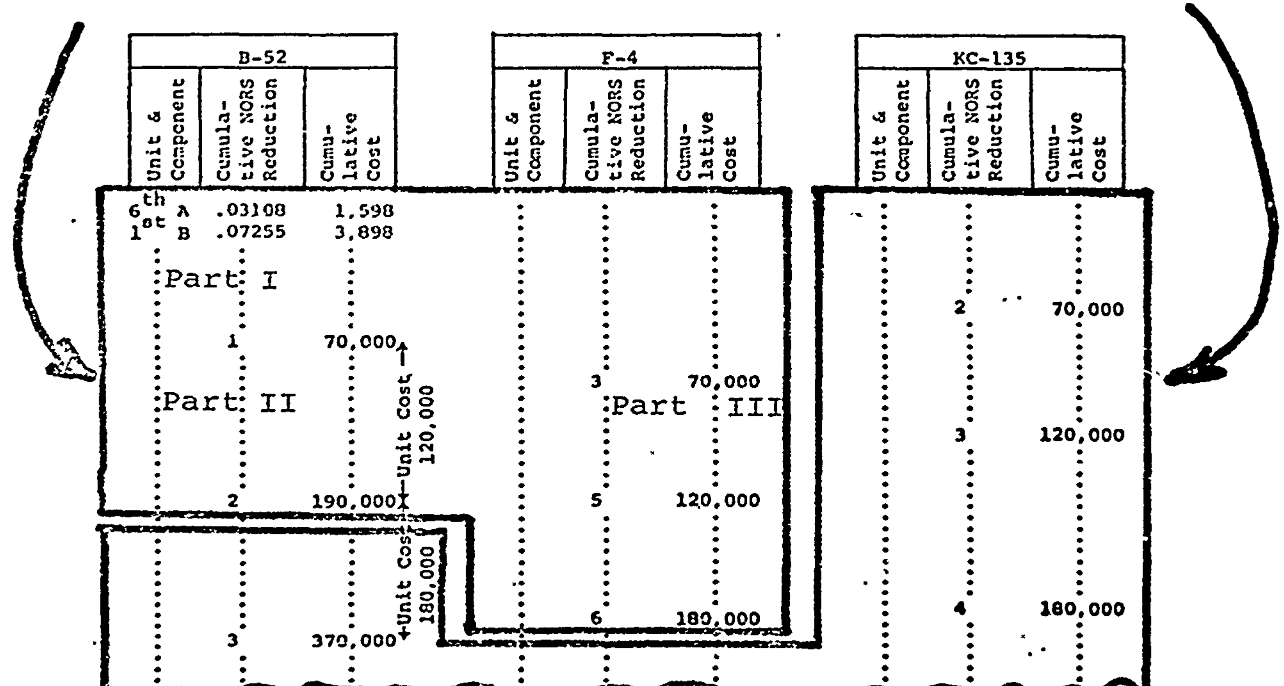

Figure 8

E. EXTENSION OF THE MODEL TO CONSIDER COMPONENTS WHICH DO NOT CAUSE A NORS CONDITION

Components classified in categories C and D in the Air Force Recoverable Item Classification System cannot cause an aircraft to be NORS (see page 5). Instead, when these components are in need of repair and a spare is not readily available, the aircraft will be classified as NFE (not fully equipped). If the component is classified C, it may cause the aircraft to be in a serious NFE condition. Those classified D cause a minor NFE condition. Less than 15% of all Air Force recoverable components are classified C or D. (We are ignoring a few components which have been classified E because they are primarily obsolete items which would not be procured.)

Below we describe a method of merging NFE-causing recoverable component units into the list of NORS-causing units generated by models described in II.C and II.D above.

1. <u>General Approach</u>

For each aircraft type, high level planners will be asked questions such as the following. "Would you rather have two aircraft (of the type being considered) in serious NFE condition or one NORS aircraft?" If the answer is, "We would rather have two serious NFE airplanes," the next question would be, "Would you rather have three of those aircraft in a serious NFE condition or one NORS aircraft?" Assume that the preference is "three serious NFE aircraft." The next question would then be, "Would you rather have four in a serious NFE condition or one NORS?" (We are searching for a point of indifference.) If he answers that four to one is about a tossup, an equivalence has been established as follows. Four serious NFE aircraft of the type being considered are equal to one NORS.

Similar questions are asked regarding aircraft in a minor NFE status until a three-way equivalency[1] is established for the aircraft type being considered, e.g., one NORS equals four serious NFE equals ten minor NFE.

2. Merging Procedure

The LMI NORS Model is used to generate shopping lists for serious NFE items and minor NFE items. The procedure is the same as the one used for NORS-causing items. For instance, the serious NFE list would be sorted in such a way that if you procure units in the order of the list, you can be assured that at each step you have reduced the expected number of serious NFE aircraft the maximum amount possible with the amount of money spent to that point. When the NORS, serious NFE, and minor NFE lists have been generated for one aircraft type, they can be merged using the equivalence information obtained by questioning the high level planners. The merging of the lists into one integrated list can be accomplished by using the procedure described in Appendix 3, Section D.

F. COMPONENTS USED ON MORE THAN ONE AIRCRAFT TYPE

Introduction

The models discussed in II.C and II.D above assume that a component is single-use, i.e., used on only one aircraft type. In reality this assumption is not true for many components.

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[1]Since the importance of secondary missions varies from aircraft type to aircraft type the equivalence or tossup points will vary from aircraft type to aircraft type.

Based on analysis to date the LMI model assumes that equivalency ratios are essentially constant over the limited range of NORS rates of interest. This matter will be given additional study and the assumption either verified or the model modified.

Thus a procedure is required to interpret a multi-use component as if it were a number of different single-use components. The procedure proposed below allows this form of interpretation. The procedure is explained by use of a simple hypothetical example.

1. <u>Multi-Use Component Procedure</u>

Assume component C is used on aircraft types T and $\overline{T}$.

<u>Steps</u>

1) Assume (temporarily) that component C can be partitioned into two different components as follows.

   CT used only on aircraft type T

   $C\overline{T}$ used only on aircraft type $\overline{T}$

2) Using historical and planning data pro rate the projected demand rates for component C, DC, among the two hypothetical components CT and $C\overline{T}$ to obtain DCT and $DC\overline{T}$.

3) Pro rate the projected initial stock level[1] of component C to the hypothetical components CT and $C\overline{T}$ in the ratio of the projected demand rates DCT and $DC\overline{T}$.

4) Determine other data required to run the METRIC LMI Model for the hypothetical component CT and $C\overline{T}$, e. g., base repair time. The data required are listed on pages 9 and 10.

5) Run the METRIC LMI and LMI NORS Models for all components for aircraft types T and $\overline{T}$ as though CT and $C\overline{T}$ were different single-use components.

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[1] See page 14.

6) Merge the shopping lists obtained for all aircraft types, including T and $\overline{T}$, in accordance with the procedure described in II.D.

7) At each place in the shopping list where a CT or $C\overline{T}$ unit is listed change the component name to C and set the corresponding stock level (unit number) equal to the sum of the stock levels of CT and $C\overline{T}$ to that point.

2. <u>Errors Introduced by the Multi-Use Component Procedure</u>

Stock levels for multi-use components established by the above procedure can be too large or too small when compared with stock levels for single-use components. In many instances the factors which tend to make the multi-use levels too large will be offset by the factors which make the levels too small. Some of the errors which may be introduced by the multi-use component procedure are discussed below.

- Factors which Can Cause the Multi-Use Component Levels to Be Too High

When aircraft types T and $\overline{T}$ are supported by different bases but by the same depot, stock levels computed by the recommended procedure may be slightly high. This is due to the fact that the computed depot level stocks will reduce end use back orders more than the sum of the reductions computed by METRIC for the hypothetical components CT and $C\overline{T}$.

When the aircraft types are supported by the same bases the error will be even larger since both the depot and base levels will provide more protection than that computed by METRIC.

- Factors which Can Cause the Multi-Use Component Stock Levels to Be Too Small

Assume that aircraft type $\overline{T}$ in an optimal plan would have a much lower NORS rate than would aircraft type T. (It is

more important to support aircraft type $\bar{T}$ than it is to support aircraft type T.) Assume also that the optimum stock levels for a hypothetical component CT and $C\bar{T}$ were as follows.

CT stock level = 10

$C\bar{T}$ stock level = 50

Also assume that the demand rates DCT and $DC\bar{T}$ are approximately equal and the aircraft types T and $\bar{T}$ are supported by the same bases.

Assume also that the system-wide stock level for component C is set equal to 60. In this case the system-wide stock level of component C would tend to be too low since in practice it is almost impossible to prevent aircraft type T from drawing spares which were assumed by the model to be reserved for aircraft type $\bar{T}$. In reality, aircraft type T would receive more support than called for in an optimal plan while aircraft type $\bar{T}$ would receive less. But aircraft type $\bar{T}$ is more important than aircraft type T. Thus the losses in military readiness would outweigh the gains.

3. Summary

Considering the current state of the METRIC Program and the policies governing the issue of recoverable components to end users, it does not appear practical to generate a more sophisticated model for multi-use components than that described in Section 1 above. The procedure will generate stock levels sometimes too large, sometimes too sma and sometimes about optimal. But that procedure will take into consideration the military essentiality of the components. Thus it should be superior to current procedures.

G. MODIFICATIONS TO MAKE THE MODEL MORE RESPONSIVE TO THE REAL AIR FORCE ENVIRONMENT

The LMI NORS Model as described thus far disregards a number of factors which affect NORS rates to some degree. Paramount are

1. A NORS condition can be caused by consumable parts not covered by the METRIC system.

2. A NORS condition can in certain instances be eliminated by cannibalization, expedited repair, or expedited shipment.

3. Judicious planning of scheduled maintenance can decrease NORS rates by exploiting the time an aircraft would otherwise be NORS to accomplish required maintenance.

4. Condemnations[1] will increase NORS rates when deliveries to replace condemnations lag condemnations. On the other hand, when such deliveries are received faster than condemnations occur, additional spares are generated. Such spares can be used to reduce NORS rates.

The overall effect of the factors not considered directly by the LMI NORS Model can be measured. Appropriate correction parameters can then be introduced into the LMI NORS Model so that predictions will more accurately reflect the actual operating environment. The recommended procedure is described below.

For each aircraft type the LMI NORS Model should be run to predict the current NORS rate considering the existing spares

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[1]When it is determined that a removed recoverable unit will not be repaired, that unit is condemned (discarded).

levels for all recoverable components. The computed NORS rates should then be compared to the actual NORS rates to generate correction parameters to be used for future predictions. The process should be repeated each six months and additional correction parameters and refinements to the LMI NORS Model generated.

III. CONSIDERATION OF MILITARY ESSENTIALITY IN THE ESTABLISHMENT OF REORDER POINTS FOR HIGH COST CONSUMABLE PARTS

A stock of spare recoverable components can be thought of as a self-replenishing pool such that every demand on the pool results in a subsequent replenishment to the pool. The quantity of units on hand (ready for issue) and on order (being repaired), minus the quantity of unfilled end use demands, remains constant so long as there are no condemnations and no deliveries of new units into the system.

One standard doctrine for stocking consumable parts is analogous to a self-replenishing pool. Under this doctrine, a reorder point, R, is established for each item. When the quantity of the item on hand and on order drops to R, a requisition is initiated for an additional quantity of the item. The quantity requisitioned is an economic order quantity (EOQ) computed in accordance with the Wilson formula.[1] If the EOQ equals one and the quantity on hand and on order drops to R, then the last demand and every demand thereafter will trigger a requisition and a subsequent replenishment. Just as in the case of a self-replenishing pool of recoverable components the quantity of units in the system and on order minus the quantity of unfilled end use requisitions remains a constant (R+1). Thus, there is no conceptual difference

[1] For derivation and background see G. Hadley and T. M. Whitin, Analysis of Inventory Systems. (Englewood Cliffs, N. J.: Prentice-Hall, Inc. [1963]), Section 2.

between a stock level of spare recoverable components of size R+1 and a consumable stock having a reorder point equal to R and an EOQ equal to one.

In view of the above, the methods described in Section II can be adapted to establish optimum system-wide reorder points for any high cost low demand consumable part whose procurement EOQ is equal to one.

IV. CONSIDERATION OF MILITARY ESSENTIALITY IN THE DEVELOPMENT OF DISTRIBUTION PLANS

The generation of distribution plans is the process by which base and depot stock levels are established, component by component, considering available stocks. The distribution problem differs from the procurement problem as follows:

- The procurement[1] problem involves the allocation of available funds, the principal constraint, among many components used on many aircraft types. Thus in the procurement problem different components and different aircraft types must be considered simultaneously and the interaction among, and relative essentiality of those components and aircraft types should be taken into consideration.
- In the distribution problem there is no constraint related to (across) more than one component. Instead, there is a separate constraint (the available stock) for each component. Furthermore the distribution problem is continually being re-solved for those components where the available stocks have changed significantly. Thus in the distribution problem components can be, and should be considered

[1]See footnote, p. 6.

one at a time and it is reasonable to ignore the interaction among, and the relative essentiality of different components.

Under the METRIC System distribution plans can be generated, component by component, with the primary objective of minimizing expected back orders for the component being considered. There is no reasonable way, and no pressing need, to make trade off decisions among different components. Furthermore, minimization of expected back orders, component by component, will provide a solution which gives the lowest NORS rates possible considering the data now being collected or feasible of collection in the near future. Accordingly, we recommend no changes to the current METRIC approaches for developing distribution plans.

APPENDIX 1

ASSISTANT SECRETARY OF DEFENSE
Washington, D. C.

Installations and Logistics DATE: 14 July 1971

TASK ORDER SD-271-159
(Task 72-3)

1. Pursuant to Articles I and III of the Department of Defense Contract No. SD-271 with the Logistics Management Institute, the Institute is requested to undertake the following task:

A. TITLE: Measurements of Military Essentiality

B. SCOPE OF WORK: Underlying each inventory or stock control policy (e.g., reorder point, safety level, reorder quantity) is an assumed value of the military essentiality or worth of the part or component to which it applies. However, in most instances objective measurements of military essentiality or worth are not made and arbitrary estimates are substituted. In some instances a cost is assigned to a stock-out. In other instances, arbitrary requisition fill percentages are applied. On the other hand, a few models have been developed which utilize objective methods of making realistic measurements of worth or military essentiality. Under this task LMI will:

1) Review methods which have been developed for measuring or estimating military essentiality or worth and determine their applicability to various classes of Air Force parts and components,

2) Evaluate the need and feasibility of developing additional methods,

3) Recommend methods to be used for determining the relative essentiality of Air Force parts and components. Special attention will be given to insuring consistency with DODI 4140.39, "Procurement Cycles and Safety Levels of Supply for Secondary Items," in which provisions are made and policy set forth for the use of an essentiality function by the Military Departments and the Defense Supply Agency.

2. SCHEDULE: The task will be completed and a final report will be submitted by 31 July 1972.

APPROVED [signature]

ACCEPTED Wm. Fiuau

DATE 14 July 1971

Appendix 2

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Appendix 3

TECHNICAL DEVELOPMENT

INTRODUCTION

In this appendix we describe and justify the mathematical models and logic used in METRIC-LMI and LMI NORS models. In Section A we describe, but do not justify the mathematical formulations used in the models. In Sections B and C the program logic of the METRIC-LMI and LMI NORS models is described. In Section D technical justifications for the models used and the mathematical proofs are presented.

A. MATHEMATICAL FORMULATION

For each component type the METRIC-LMI Model computes

XBO_n	The expected number of end use back orders given any quantity, n, of spares in the system.
$RXBO_n = XBO_{n-1} - XBO_n$	The expected back order reduction which would be obtained by addition of the n^{th} spare unit.

Those values together with other data described on the following page are used in the LMI NORS Model.

The mathematical formulation for the LMI NORS Model is presented on pages 44, 45, 46, and 47. For clarity the main mathematical formulation is presented by use of a simplified hypothetical example.

NOTATION & DEFINITIONS

A/C TYPE IS FIXED

N QUANTITY OF A/C (OF TYPE BEING CONSIDERED) IN THE SYSTEM

i THE COMPONENT BEING CONSIDERED. SPECIFIC COMPONENT TYPE DENOTED BY A, B, C, ETC.

n THE QUANTITY OF SPARES (OF THE COMPONENT BEING CONSIDERED) IN THE SYSTEM. SPECIFIC QUANTITIES DENOTED BY 0,1,2,3, ETC.

C_i UNIT COST OF COMPONENT i

QPA_i QUANTITY OF COMPONENT i ON ONE OPERATIONAL A/C; E.G., QPA_B MIGHT BE 2 (BRAKES)

$XBO_{i,n}$ EXPECTED QUANTITY OF END-USE BACK ORDERS (COMPONENT i) WHEN THE QUANTITY OF SPARES IN SYSTEM IS n; E. G., $XBO_{A,2}$ MIGHT BE 9.3601

= QUANTITY COMPUTED BY METRIC-LMI PROGRAM

$q_{i,n}$ PROBABILITY THAT A RANDOM A/C AT A RANDOM POINT IN TIME WILL NOT HAVE ANY COMPONENTS OF TYPE i MISSING BECAUSE OF SUPPLY - ASSUMING QUANTITY OF SPARES IN SYSTEM IS n

$$= \left(1 - \frac{XBO_{i,n}}{QPA_i \cdot N}\right)^{QPA_i}$$

Q_S PROBABILITY THAT A RANDOM A/C AT A RANDOM POINT IN TIME WILL <u>NOT</u> BE NORS GIVEN S, A PARTICULAR COMBINATION OF COMPONENT STOCK LEVELS; E.G., 6 ALTIMETERS, 4 AIR SPEED INDICATORS, 14 BRAKES, ETC.

$N \cdot Q_S$ EXPECTED QUANTITY OF <u>NOT</u> NORS A/C IN SYSTEM GIVEN S

$N - N \cdot Q_S$ EXPECTED QUANTITY OF NORS A/C IN SYSTEM

NORS MODEL
(WITH SIMPLIFIED EXAMPLE)

A/C Type is Fixed

Qty of A/C in System, N, is 100
Only 4 components (A,B,C and D) are on an A/C

STARTING INVENTORY OF SPARES (Initial s)

Component	Qty	Expected Back Orders If No More Procured	(Qty per A/C)	Probability That A Random A/C At A Random Pt. in Time Will Not Be Missing any of the Components In Question Because Of Supply
A	2	$XBO_{A,2} = 9.3601$	1	$q_{A,2} = .906399$
B	10	$XBO_{B,10} = 6.2193$	1	$q_{B,10} = .937807$
C	7	$XBO_{C,7} = 4.9166$	2	$q_{C,7} = .951438$
D	20	$XBO_{D,20} = 1.3221$	1	$q_{D,20} = .986779$

Probability that a random A/C at a random point in time will Not be NORS if no more spares are procured is

$$Q_s = q_{A,2} \cdot q_{B,10} \cdot q_{C,7} \cdot q_{D,20} = .798048$$

(Giving $100 \cdot Q_s = 79.8048$ = Expected Qty of A/C Not NORS)

Suppose that if one more of component A (the 3rd one) is procured the expected back orders (of component A) will be reduced from 9.3601 to 8.4210.

$$\text{i.e., } XBO_{A,2} = 9.3601$$
$$XBO_{A,3} = 8.4210$$

Then $q_{A,3} = .915790$ and

$$Q_{s+3^{rd}\,A} = q_{A,3} \cdot q_{B,10} \cdot q_{C,7} \cdot q_{D,20} = .806325$$ (Giving 80.6325 A/C not NORS)

$$\frac{Q_{s+3^{rd}A}}{Q_s} = \frac{q_{A,3} \cdot q_{B,10} \cdot q_{C,7} \cdot q_{D,20}}{q_{A,2} \cdot q_{B,10} \cdot q_{C,7} \cdot q_{D,20}} = \frac{q_{A,3}}{q_{A,2}} = 1.01037 = \text{NORS IMPROVEMENT FACTOR}$$

or $$\boxed{Q_{s+3^{rd}\,A} = \frac{q_{A,3}}{q_{A,2}} \cdot Q_s} = .806325$$

GENERALIZED MODEL TO COMPUTE EXPECTED QTY OF NORS A/C AT EACH STEP OF A SHOPPING LIST

$$Q_{AFTER} = \frac{q_{i,n}}{q_{i,n-1}} \cdot \text{multiplied by } Q_{BEFORE}$$

Q_{AFTER}	$= \frac{q_{i,n}}{q_{i,n-1}}$	multiplied by	Q_{BEFORE}
↓	↓		↓
NOT NORS PROBABILITY AFTER n^{th} UNIT OF COMPONENT i IS OBTAINED	= NORS IMPROVEMENT FACTOR	multiplied by	NOT NORS PROBABILITY BEFORE n^{th} UNIT OF COMPONENT i IS OBTAINED
↓	↓		↓
EXPECTED QTY OF NOT NORS A/C AFTER n^{th} UNIT OF COMPONENT i IS OBTAINED	= NORS IMPROVEMENT FACTOR	multiplied by	EXPECTED QTY OF NOT NORS A/C BEFORE n^{th} UNIT OF COMPONENT i IS OBTAINED

SORTED LIST TO MINIMIZE NORS FOR DIFFERENT FUND ALLOCATIONS

(ONE A/C TYPE)

(100 A/C IN SYSTEM)

STARTING POINT: $Q_S = .866271$ = Probability a random A/C at random time will not be NORS given qtys of components on hand and on order; i.e., no new procurement

$100 \cdot Q_S = 86.6271$ = Expected qtys of A/C not NORS with no new procurement

Unit	$q_{i,n-1}$	$q_{i,n}$	$\frac{\log q_{i,n} - \log q_{i,n-1}}{C_i}$	Expected qty of A/C not NORS after unit is obtained	Cumulative NORS Reduction	Cumulative Cost
2nd F	.962610	.968413	$2.6102 \cdot 10^{-6}$	87.1493	.5222	1000
1st G	.980000	.982720	$2.4000 \cdot 10^{-6}$	87.3912	.7641	1500
3rd F	.968413	.972890	$2.0031 \cdot 10^{-6}$	87.7952	1.1681	2500
21st D	.991200	.991655	$1.9943 \cdot 10^{-6}$	87.8355	1.2084	2600

The list is sorted in descending order of this value.

$$Q_{after} = \frac{q_{F,2}}{q_{F,1}} \cdot Q_{before} = \frac{.968413}{.962610} \cdot .866271 = .871493$$

$$Q_{after} = \frac{q_{G,1}}{q_{G,0}} \cdot Q_{before} = \frac{.98272}{.98000} \cdot .871493 = .873912$$

B. METRIC-LMI PROGRAM LOGIC

The purpose of the LMI addition to the METRIC Program is to find for any quantity of spare units of a component in the system the combination of depot and base levels that minimizes the expected back orders. Using those values it then computes the expected back order reductions which would be obtained by the addition of each unit to the system.

METRIC is used to compute the expected back orders for all combinations of depot and base levels. We will designate the expected back orders as B_{ij} where

i spares are allocated to a depot and

j is the sum of all spares allocated to bases.

The first algorithm in the LMI addition to METRIC finds the smallest B_{ij} for any n such that $n = i + j$ (n is the quantity of spares in the system as a whole). In essence this algorithm finds the smallest value along each diagonal in the following table.

QUANTITY ALLOCATED TO BASES

QTY ALLOCATED TO DEPOT	0	1	2	3	4	.	.	.
0	B_{00}	B_{01}	B_{02}	B_{03}	B_{04}	.	.	
1	B_{10}	B_{11}	B_{12}	B_{13}	.	.	.	
2	B_{20}	B_{21}	B_{22}	.	..	.		
3	B_{30}	B_{31}	.	.	.	.		
4	B_{40}	.	.	.	.	.		
.	.	.	.	.	.	.		
.	.							
.								

In order to accomplish this step efficiently using a minimum of computer storage the program generates the following table using an iterative process.

QTY IN SYSTEM (n)	EXPECTED BACK ORDER (XBO_n)	DEPOT LEVEL (i)
0	XBO_0	0
1	XBO_1	0 or 1
2	XBO_2	0,1 or 2
.	.	.
.	.	.
.	.	.
n	XBO_n	0,1,..., or n
.	.	.
.	.	.
.	.	.

When the process is completed the locations (XBO_n) in the expected back order column in the above table will contain the smallest value for the corresponding system stock quantity. The depot level column will contain the depot level corresponding to that minimum expected back order value. Logic of the algorithm is as follows:

- All values XBO_n are set equal to zero.
- All values in the depot level column are set equal to zero.
- Input data regarding a component is read into METRIC. (The data required is described on pages 9-10.)

- Additional data regarding the component in question are read in. Those data are

  N = Number of aircraft (of type being considered) to be supported

  QPA = Quantity (of component being considered) on one operational aircraft

  CST = Unit cost of component being considered

- METRIC sets i (depot level) equal to zero and computes the expected back orders for each value of base levels, i.e., it computes all values $B_{\emptyset j}$ greater than some specified constant. (In practice the constant is a number close to Ø.) As this computation is progressing XBO_n is set equal to $B_{\emptyset j}$. (During this step n equals j because i equals Ø.)

- METRIC raises the depot stock level (i) by one, computing all expected back order values. If, but only if, computed $B_{ij} <$ recorded XBO_n then

  XBO_n is set = new B_{ij} and

  Corresponding depot level i is set = new i

  For instance, if $B_{12} < XBO_3$, then XBO_3 is set equal to B_{12} and the corresponding depot level entry is set equal to 1.

- The preceding step is continued until i equals the maximum j for which a B_{ij} was computed.

For each quantity n for each component the following fields are written on an output record

- Component name
- i = depot stock level
- n - i = sum of base stock levels
- XBO_n = minimum expected back orders, given n.
- $RXBO_n$ = back order reduction $(XBO_n - XBO_{n-1})$ obtained by n^{th} unit
- Unit cost

C. LMI NORS PROGRAM LOGIC

Further computations are made on the METRIC-LMI output data to produce intermediate records with the following fields for each component.

ID	= Component Name
n	= Total spare units in the system
CST	= Unit Cost
XBO_n	= minimum expected back orders, given n
q_n	$= \left(1 - \dfrac{XBO_n}{N \cdot QPA}\right)^{QPA}$
q_{n-1}	$= \left(1 - \dfrac{XBO_n + RXBO_n}{N \cdot QPA}\right)^{QPA}$
$LN(q_n/q_{n-1})$	= Natural logarithm of q_n/q_{n-1}
ADJ $LN(q_n/q_{n-1})$	= Original $LN(q_n/q_{n-1})$ adjusted by the averaging routine (described below and in Section D) to assure monotonically decreasing values
FLAG =	= "Yes" or "No." Indicates a legitimate step down a shopping list generated by the averaging routine
QPA	= Quantity per aircraft

Before running the averaging routine the value of ADJ LN (q_n/q_{n-1}) is set equal to LN (q_n/q_{n-1}). The averaging routine adjusts the values of ADJ LN (q_n/q_{n-1}) so that they are monotonically decreasing as n increases (within one component) and flags legitimate steps in a shopping list. On the following page is a flow chart showing the logic used to average. Section D of this appendix provides justification of the method used and a mathematical proof.

The program also computes the quantity of expected NOT-NORS aircraft (NQ_s) as follows. For each component i used on an aircraft the projected stock level s_i and the component identification are read in. "NOT NORS" probability values q_{i,s_i} are multiplied to obtain

$$Q_s = \prod_i q_{i,s_i}$$

and

$$NQ_s$$

The entire data set for an aircraft type, in order by units within component ID, is then sorted by $\dfrac{\text{ADJ LN } (q_n/q_{n-1})}{\text{CST}}$ to produce the following report:

Component ID

Unit

ADJ LN (q_n/q_{n-1})/CST

Expected AC NOT NORS = $\dfrac{q_n}{q_{n-1}} \cdot$ Expected AC NOT NORS after procurement of all items higher on the list

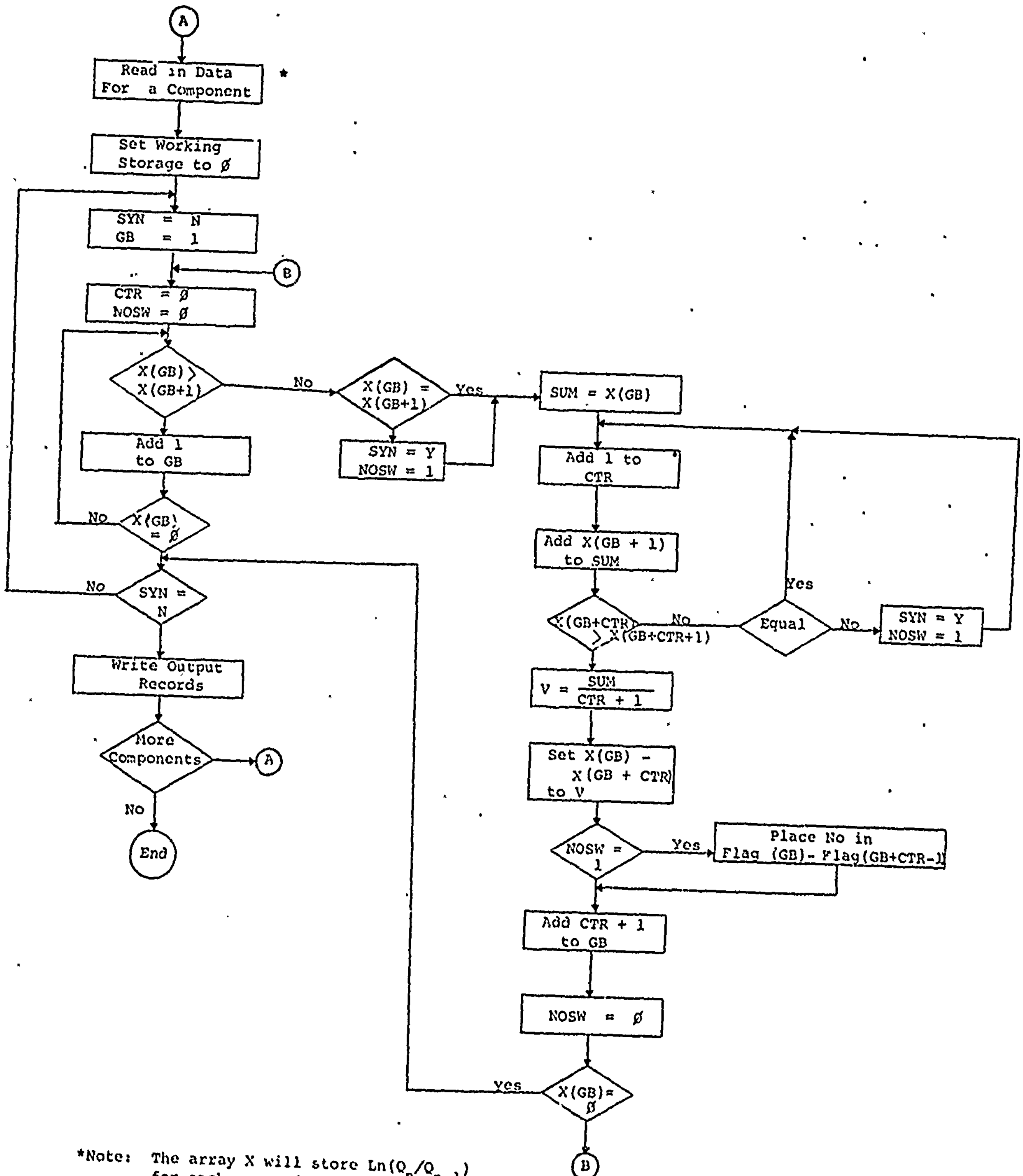
Cumulative NORS Reduction

Unit Cost

Flag ("Yes" or "No") Indicates whether entry is a legitimate stopping point

Cumulative Cost

AVERAGING REVERSALS



*Note: The array X will store $Ln(Q_n/Q_{n-1})$ for each unit of a component in location X_n.

D. TECHNICAL JUSTIFICATION AND PROOFS

1. Back Orders Versus NORS Minimization

Many components on aircraft are classified and managed as repairables or recoverables. Any time a failure of such a component occurs the failed component is removed and repaired. When spares are not available the aircraft must do without, or remain grounded, until a repaired unit is received. For more effective maintenance, stocks of spares are utilized.

The problem addressed in this section concerns the quantities of spare repairable components to procure when funds are limited. In particular comparisons are made of two different methodologies and the practical consequences of their use: the one used by the METRIC system and the one proposed. The METRIC system minimizes the expected number of back orders. The system being proposed minimizes the expected number of NORS aircraft.

For simple exposition of the different characteristics of the two systems the analysis is restricted to problems satisfying the following simplifying assumptions concerning the aircraft in the system and the class of (repairable) components.

(A1) The number of aircraft in the system is constant

(A2) All aircraft in the system are of the same type

(A3) On each aircraft there is exactly one unit of each component

(A4) Each unit is necessary to keep the aircraft operational

Thus a back order generated by an aircraft for a component of this class signifies that the aircraft is NORS (not operationally ready while waiting for supply). Of course the same NORS aircraft may have generated a number of different back orders.

Section 1.1 lists information derived from available statistical and planning data (demands assumed to be independent, repair times, etc.). Sections 1.2 and 1.3 briefly review and compare the two methods. Section 1.4 discusses a major consequence.

1.1 Notation and preliminaries

N = total number of A/C of the given type

K = total number of distinct (reparable) components on a plane of the given type; we identify these various components by numbers 1, 2, .. through K;

s_i = stock level of component #i = the number of spares of component #i in the system

$B_i(s_i)$ = expected number of back orders of component #i, when the stock level of component #i is s_i

$p_i(s_i) = \frac{B_i(s_i)}{N}$ = expected number of component i back orders per aircraft when the stock level of component #i is s_i

= the probability that on a random plane at a random point in time the component #i will be missing <u>and</u> out of stock (i.e. component #i has been removed from the plane for repair <u>and</u> there is no spare available), when the stock level of component #i is s_i

$q_i(s_i) = 1 - p_i(s_i)$ = the probability that such will not occur

We frequently write n for s_i, thus $q_i(n) = q_i(s_i)$, and also $q_{i,n}$ for $q_i(n)$.

1.2 Minimizing expected back orders

To minimize the expected back orders is the same as minimizing the sum of all $B_i(s_i)$; this is the same as minimizing the expected back orders per aircraft, i.e., the sum of all $p_i(s_i)$. This in turn is the same as maximizing the sum of all $q_i(s_i)$ since that sum equals to K minus the sum of all $p_i(s_i)$.

Thus the problem of minimizing expected back orders subject to a budget constraint can be stated as (is equivalent to):

$$(1.2.1)\quad \begin{cases} \text{Maximize } V = q_1(s_1) + q_2(s_2) + \ldots + \ldots = \sum_i q_i(s_i) \\ \text{subject to } C_1 s_1 + C_2 s_2 + \ldots + \ldots = \sum_i C_i s_i \leq b \end{cases}$$

where C_i = the cost of 1 unit of component #i

The solution to this problem involves finding the stock levels of the various components that satisfy the two conditions: their total cost does not exceed the given budget and V is a maximum.

The method of solution (omitting technical details--to be discussed in Section 2) consists of building up these stock levels step by step. At each step, just one unit is added to the stock level of some component (for instance: the 14th altimeter to the 13 we have, or the 9th engine to the 8 we have). But which component should be added at a given step? Assume first all components had the same price. Then obviously the component to choose would be the one that would achieve the greatest increase in the value of V. That increase is very simple to

compute for each component: since V is a sum (of the q_i's), the increase in V is simply the increase in the affected summand (in the q_i for component #i). Thus, the increase in V, achieved by adding the m-th unit of component #i would equal

$$q_i(m) - q_i(m-1)$$

Since, however, different components will generally have different prices, the criterion for choosing the component to be added at a given step would be the increase in the value of V per unit cost, that is:

$$\frac{q_i(m) - q_i(m-1)}{c_i}$$

For instance, the choice between adding the 14th altimeter versus the 9th engine would be made according to the greater of the two values

$$\frac{q_A(14) - q_A(13)}{\text{cost of 1 altimeter}}, \quad \frac{q_E(9) - q_E(8)}{\text{cost of 1 engine}}$$

1.3 Minimizing NORS

Denote by s the stock level of spares for all components, thus

$$s = (s_1, s_2, \ldots, s_i, \ldots, s_K).$$

To minimize the expected number of NORS aircraft is the same as maximizing the expected number of not-NORS aircraft. If

$Q(s)$ = the probability that a random aircraft at a random time point will not be NORS, when the stock level is s,

then the expected number of not-NORS aircraft = $N \cdot Q(s)$, and to maximize $N \cdot Q(s)$ is equivalent to maximizing $Q(s)$.

Now, an aircraft is not-NORS if and only if no out-of-stock component is missing on it. Therefore $Q(s)$ = the product of all $q_i(s_i)$, that is

$$(1.3.0) \quad Q(s) = q_1(s_1) \cdot q_2(s_2) \ldots . q_i(s_i) \ldots \cdot q_K(s_K) = \prod_{i=1}^{K} q_i(s_i)$$

and the problem is to

$$\begin{array}{c} \text{NORS} \\ \text{problem} \\ \text{(1st formulation)} \end{array} \left\{ \begin{array}{l} \text{Maximize } W = q_1(s_1) \cdot q_2(s_2) \ \ldots \ q_K(s_K) \\ \text{subject to } C_1 s_1 + C_2 s_2 + \ldots + C_K s_K \leq b \end{array} \right.$$

This formulation expresses the problem of minimizing expected NORS in terms of the same entities (i.e. the C_i, s_i and q_i) as those used in the formulation given for minimizing expected back orders.

The two problems differ in one and only one feature: there, the value to be maximized was the <u>sum</u> of the $q_i(s_i)$, here it is the <u>product</u> of the same $q_i(s_i)$. This is quite a difference in objectives.

As illustration, in a general sense, of the different nature of maximization of the sum of numbers versus maximization of their product, consider the following example: In a corner of your back yard, where the back fence of your property meets the side fence, you want to fence off a rectangular piece to serve as a vegetable garden. You decide to use two fence types

that will match the back and the side fence they will hook on to; they will cost 2 and 3 dollars a foot. You want to spend at most \$60, and achieve a maximal area for your vegetable garden. That is, if you buy x feet at \$2 and y feet at \$3 you want to maximize $A = xy$

$$\text{subject to } 2x + 3y \leq 60$$

The solution is $x = 15$, $y = 10$. Thus you buy 15 feet at \$2 and 10 feet at \$3 (getting an area of 150 square feet). If for some other purpose (say for just an ornamental fence design in the middle of your lawn) you want to maximize their total length $L = x+y$ subject again to $2x + 3y \leq 60$, the obvious "solution" is $x = 30$, $y = 0$

While it is now clear that our present problem--of minimizing NORS, which is our real objective--is altogether different from the back order problem, we still would like to solve it by means of a step-by-step build up of the supply levels adding at each step a unit of that component to the given supply level which achieves the greatest increase in the value of W per unit cost.

However that increase per unit cost is no longer as simple to compute as it was in the back order problem. There, where V was the sum of the $q_i(s_i)$, we could say that the increase in V achieved by adding a unit of a specific component is the same as the increase that component achieves in its own <u>summand</u> $q_i(s)$. Here it is not true that the increase an added component achieves in the value of W is the same as the increase in its own <u>factor</u> $q_i(s)$.

For this reason we now seek still another (equivalent) formulation of the NORS problem.

For this purpose we use the fact that the logarithm of a quantity increases with the quantity, and conversely. Therefore: to maximize W is equivalent to maximizing the logarithm of W.

But since the logarithm of a product equals the sum of the logarithms of each factor, the NORS problem now reads

$$(1.3.1) \quad \begin{cases} \text{Maximize } F = \text{Log}\, W = \text{Log}\, q_1(s_1) + \text{Log}\, q_2(s_2) + \ldots + \text{Log}\, q_K(s_K) \\ \text{subject to} \quad C_1 s_1 + C_2 s_2 + \ldots + C_K s_K \leq b \end{cases}$$

Now the NORS problem is of precisely the same <u>form</u> as the back order problem, in the sense that the quantity to be maximized (namely F) is again a <u>sum</u> (though not of the $q_i(s_i)$ but of the Log $q_i(s_i)$). Consequently in the solution method of step-by-step build up of the stock level s, the increase in F an added component will achieve per unit cost is again simple to compute: it is the increase it achieves, per unit cost, in the value of <u>its own summand</u>. Hence the increase depends only on the size of the component stock level. Thus if it is the nth unit of component #i that is being added, the increase per unit cost will be

$$(1.3.2) \quad \Delta F = \frac{\text{Log}\, q_i(m) - \text{Log}\, q_i(m-1)}{C_i} = \gamma_i(m)$$

and at each given step the component to be added is the one for which this computed value is largest.

Thus, to solve the NORS problem, one must merely compute these values, $\gamma_i(m)$, for every unit of every component and sort in decreasing order.

Of course, if so desired, now that we have the formula for the values to serve as the sort (choice) criterion, we can easily get rid of the logarithms. Obviously we have

$$(1.3.3) \qquad \Delta F = \frac{\mathrm{Log}\,\dfrac{q_i(m)}{q_i(m-1)}}{C_i} = \mathrm{Log}\left[\frac{q_i(m)}{q_i(m-1)}\right]^{\frac{1}{C_i}} = \gamma_i(m)$$

Hence, omitting "Log" we can also use the values

$$(1.3.4) \qquad \left[\frac{q_i(m)}{q_i(m-1)}\right]^{\frac{1}{C_i}} = \lambda_i(m)$$

as our sort criterion, since they are in the same magnitude relations as their logarithms.

To obtain an interpretation for the meaning of $\lambda_i(m)$, denote

$A = (i,m) =$ the m-th unit of component i,

$\lambda(A) = \lambda_i(m)$, $\quad C(A) =$ cost of $A = C_i$,

and in (1.3.0) set $s_i = m-1$. Then conclude from (1.3.0) and (1.3.1) that

$$(1.3.5) \qquad Q(s + A) = Q(s)\cdot \frac{q_i(m)}{q_i(m-1)} = Q(s)\left[\lambda(A)\right]^{C(A)}$$

Thus, the effect--on the function Q(s)--of adding the unit A to the stock level s, is to multiply Q(s) by the factor $\left[\lambda(A)\right]^{C(A)}$.

This suggests that

(1.3.6) $\lambda(A)$ can be interpreted as the improvement factor per unit cost; or, in other words: as the factor by which $Q(s)$ would be multiplied if one-dollar's-worth of unit A were added to the stock level s.

The last statement in (1.3.6) can symbolically be written as:

$$Q\left(s + \frac{A}{C(A)}\right) = Q(s)\ \lambda(A) \tag{1.3.7}$$

The increase per unit cost is given by

$$Q\left(s + \frac{A}{C(A)}\right) - Q(s) = Q(s)\ \left[\lambda(A) - 1\right] \tag{1.3.8}$$

1.4 Consequences

Minimization of back orders gives too much preference to low cost items when compared to minimization of NORS.

Example. Assume only two components.

H = a high-cost component, H costs \$2,000	Unit of cost
L = a low-cost component, L costs \$100	= \$100

and that both components have the same mean demand, the same repair times, etc., such that for all n

$$q_{H,n} = q_{L,n};\ q_{H,n+1} - q_{H,n} = q_{L,n+1} - q_{L,n}$$

Assume further that the differences above decrease monotonically when n increases, and that an optimal stock level that minimizes back orders is

n units of H and m units of L

Then:

$$\frac{q_{H,n+1} - q_{H,n}}{20} = \frac{D}{20} = \sigma$$

$$\frac{q_{L,m+1} - q_{L,m}}{1} = d = \sigma$$

This implies

$m >> n$, $q_{L,m} >> q_{H,n}$.

The sort for NORS Minimization uses the values

$$\left[\frac{q_{H,n+1}}{q_{H,n}}\right]^{\frac{1}{20}} = W \ , \quad \left[\frac{q_{L,m+1}}{q_{L,m}}\right]^{\frac{1}{1}} = w$$

We will show

$$W > w,$$

by showing instead:

$$W^{20} > w^{20}$$

$$W^{20} = \frac{q_{H,n+1}}{q_{H,n}} = 1 + \frac{D}{q_{H,n}} = 1 + E$$

Note:

$D = 20d$

Say:

$q_{L,m}/q_{H,n} \geq 1.1$

Then $E \geq 22e$

Say $e = 5 \times 10^{-3}$

Where $e = \dfrac{d}{q_{L,m}}$

$$w^{20} = \left[\frac{q_{L,m+1}}{q_{L,m}}\right]^{20} = \left[1 + \frac{d}{q_{L,m}}\right]^{20} = (1 + e)^{20}$$

$$= 1 + 20e + \frac{20 \times 19}{2} e^2 + \ldots + < 1 + 21e$$

$$w^{20} < 1 + 21e \qquad W^{20} = 1 + E \geq 1 + 22e > w^{20}$$

The following page contains a hypothetical example illustrating the significance of differences that can exist between Back Order and NORS Minimization.

BACK ORDER MINIMIZATION Vs NORS MINIMIZATION

Hypothetical Example at Realistic Trade-Off Point

Qty of A/C in system, N, = 100

Qty per A/C, QPA, = 1 for all components considered in example

Expected NORS rate with no further procurement = .20

Expected Not NORS rate with no further procurement = .80

		High Cost Component	Low Cost Components
Unit cost	C_i	\$ 6400	\$ 100
Expected back-orders if additional unit is not procured	$XBO_{i,\ n-1}$	10.0000	0.0500
Expected back-orders if additional unit is procured	$XBO_{i,\ n}$	9.0001	0.0343
Back order reduction obtained by procurement of additional unit	$XBO_{i,\ n-1} - XBO_{i,\ n}$	.9999	0.0157
Back order reduction divided by unit cost. (This is the backorder minimization procurement choice criterion.)	$(XBO_{i,\ n-1} - XBO_{i,\ n})/C_i$	1.56234×10^{-4}	1.57000×10^{-4} choice
Probability that random A/C at random point in time will not have component missing because of supply			
a) If additional unit is not procured	$q_{i,n-1}$	.900000	.999500
b) If additional unit is procured	$q_{i,\ n}$	.909999	.999657
NORS minimization procurement choice criterion	$(\log q_{i,n} - \log q_{i,n-1})/C_i$	$.749 \times 10^{-6}$ Choice	$.682 \times 10^{-6}$

Given only \$6400 to spend, compare NORS aircraft after procuring one high cost (\$6400) component and after procuring instead 64 low cost (\$100) components

Expected NOT NORS A/C if one \$6400 component is procured = 80.8888

Expected NOT NORS A/C if 64 \$100 components are procured = 80.8081

Thus, in this hypothetical decision, an additional 8% of one A/C would be saved at no additional cost by using the NORS minimization criterion.

2. The "Shopping List" Model

The procedure for solving (the Back Order and NORS) problems (1.2.1) and (1.3.1) has been briefly described in Section 1 as a step-by-step buildup of the stock levels for the various components in accordance with the sort criteria. It has received the suggestive name "shopping list" procedure, and is best viewed as such. We can visualize the "buyer" as carrying the "shopping list" and the money (the "funding level") he has to spend. On arrival at the store, beginning at the top of the list, he buys unit by unit in the listed order until he is left with less money than the price of the next unit on the list. The units he bought constitute an optimal stock level for the money spent. Had he entered the store with less or more money, he would still have used the same shopping list--only, he would have stopped buying earlier up, or later down the list.

Thus the shopping list procedure provides the really general solution to the optimization problem--regardless of the funding level, or, in other words, for <u>any</u> funding level.

However, the claim that the stock level so obtained is indeed <u>optimal</u> requires a formal proof. To provide such proof is the objective of this section.

To that end it will be helpful to first obtain a clear understanding of the structural properties of the shopping list. These will lead to recognition of some technical details (those "omitted" in Section 1), the distinction of various possibilities, and the emergence of some difficulties that must be surmounted.

2.1 The shopping list

Keeping the picture of the buyer, we watch him assemble his shopping list.

He sits at his desk. He has just finished working on a deck of cards which he now puts on the table to his left with the other decks he has finished. From the table to his right he now takes a deck of cards to work on. The cards in the deck are printed and read as follows.

The first card: Component #17, altimeter - unit #1

The second card: Component #17, altimeter - unit #2

......

The last card: Component #17, altimeter - unit #200

(He knows he will hardly ever buy more than 100 altimeters, but just to be prepared for the unforeseen, he has decided to include twice that many units in the list.)

His work on the deck consists in computing the sort value (1.3.2) for each unit. From tables he has prepared the previous week he can read the values of $q_{i,n}$ for each unit (n) of each component (i). Thus for the first unit of component #17 he computes the <u>sort value</u> $\gamma_{17}(1)$ by the formula

$$\gamma_{17}(1) = \frac{\text{Log } q_{17,1} - \text{Log } q_{17,0}}{C_{17}}$$

and enters the resulting $\gamma_{17}(1)$ on the first card. For the second unit, he computes

$$\gamma_{17}(2) = \frac{\text{Log } q_{17,2} - \text{Log } q_{17,1}}{C_{17}}$$

and enters the resulting $\gamma_{17}(2)$ on the second card.

He continues in this manner until the deck of component #17 is completed, and then proceeds to the deck of some other component.

When his work on all the decks has been completed, his shopping list is almost ready. All that remains to be done is to rearrange the entire set of cards in the order of decreasing sort values.

In preparing for this last step it appears infeasible to arrange all the cards into a single stack, and he also experiences a "vague sense of loss" in disturbing the neat present order of the decks. So, he decides to have a duplicate made of each card in the set, and instructs his assistant to file these duplicates in a library-type card-file system according to decreasing sort values and to label the system "Shopping File." The original set is to be filed in the same type of card-file system, labeled "Component File;" here the present order is retained: first, all the units of component #1 in their natural order, then those of component #2, and so forth.

As soon as the shopping file is ready, he will instruct the assistant (a) to enter on each card in that file the cumulative cost of all units preceding and including the given one, (b) to enter the same amount on the corresponding card (i.e., same unit of same component) in the component file.

Once these entries will have been made, the cards in the shopping file will also appear ordered according to increasing cumulative cost entries (and not only according to decreasing sort values). This will enable the buyer to transfer his function to his assistant, who will operate on the following rule:

when the approved funding level is known, go to the shopping file, find the first card where the cumulative cost exceeds the funding level, and buy all units listed on the preceding cards.

While the buyer will have good reason to be happy with the simple rule he has devised, he will also realize that the buying itself will constitute quite a job and that his assistant will in turn need a few assistants to do all this buying, unit after unit of all mixed-up components. Chances are that at this point the reason for the "vague sense of loss" he has experienced earlier will suddenly break through and lead him to an alternative rule for his assistant:

(2.1.1) Rule. For each component in the component file, locate the first card on which the cumulative cost entry exceeds the budget figure, and pull the immediately preceding card; use the pulled cards to make out the order (stating the number of units for each component).

However, we have been running slightly ahead of the operation. The buyer has made some decisions and we still intend to watch their being carried out.

Filing the duplicate cards into the shopping file has just begun. After awhile the assistant enters the buyer's office with a question. "The two cards I have here carry the same sort value; I wonder which should come first in the file: component 61, unit 7, or component 11, unit 15." The buyer hesitates but finally says, "no difference."

After another while the assistant comes with a second question. "Unit 4 and unit 5 of component 38 have the same sort

value--which should be first?" The answer this time is given immediately: "The lower unit always first."

The filing appears to go smoothly for a long while. Then comes a new question. "Here I have units 7 and 8 of component 31; unit 7 has sort value .00154, unit 8 has sort value .00193--which comes first?"

This time the buyer is really puzzled. He looks at the two cards, recomputes the sort values, but gets the same results. He tells the assistant to interrupt the filing for the time being ....

We shall now leave the buyer and his assistant, but not without thanking them for the valuable experience that we now wish to analyze.

Two conclusions are immediate. First, the two files have been helpful in rounding out the picture; we expect them to be conceptually useful in our analysis. Second, none of the assistant's questions can be ignored; they all need attention and will be dealt with systematically. The reader will at least sense that had we begun this section with a direct attempt at providing the optimality proof, we would have stumbled on the same questions and thus been forced to go back and first resolve these various difficulties.

From the present view of the problem the discussion will proceed step by step, from the simplest special case to the general case (of the problem on hand), by considering the following assumptions:

(A1) All sort values are distinct

(A2) For each fixed i, $\gamma_{i,n}$ is a monotonically decreasing function of n

$\gamma_i(n)$ is monotonically decreasing if and only if

$$\gamma_i(n) \leq \gamma_i(n') \text{ whenever } n > n'$$

Section 3 discusses the case where (A1) and (A2) are both satisfied.

Section 4 assumes only (A2).

Section 5 discusses the general case.

3. The Case (A1,2)

This case makes both assumptions (A1) and (A2). Note: (A1) implies in particular that the shopping list is totally ordered, that is: if B and C are two distinct units on the list, then exactly one of them precedes the other (recall that B precedes C whenever the sort value for B is $\geq$ the one for C).

Note: (A1) and (A2) together imply that

$\gamma_i(n)$ is strictly decreasing, that is:
$\gamma_i(n) < \gamma_i(n')$ whenever $n > n'$

An initial section of the shopping list means what the name suggests: it consists of all units on the list that precede and include a given unit; or equivalently: it is a set of units from the list with the property that, with each unit it contains, it also contains all its predecessors from the list.

3.1 Optimality Proof

Let s* denote an arbitrary initial section of the shopping list, and let the cost C(s*) = h. It will be shown that

(3.1.1) Under the assumptions (A1,2):
for the given cost h, s* is a unique optimal solution of the NORS problem (1.3.1).

Explicitly the statement means: If s is any stock level such that $s \neq s^*$ and $C(s) \leq h$, then $F(s) < F(s^*)$.

For the proof note first the identity

$$q_{i,n} = q_{i,0} \cdot \frac{q_{i,1}}{q_{i,0}} \cdot \frac{q_{i,2}}{q_{i,1}} \cdots \frac{q_{i,n-1}}{q_{i,n-2}} \cdot \frac{q_{i,n}}{q_{i,n-1}} .$$

and define $\gamma_{i,0} = \dfrac{\text{Log } q_{i,0}}{C_i}$.

Then

$$\text{Log } q_{i,n} = C_i \ \gamma_{i0} + C_i \ \gamma_{i1} + \ldots + C_i \gamma_{in}$$

and hence F(s), which in 1.3.1 is of the form

$$F(s) = \text{Log } q_1(s_1) + \ldots + \text{Log } q_K(s_K)$$

can also be written in the form

$$F(s) = \sum_{(i,v)\epsilon s} C_i \ \gamma_{i,v}$$

where (i,v) is simply a short notation for "the v-th unit of component number i"

Next let t denote the set of all units common to s* and s. Then s* is the union of t and some set x; to emphasize disjointedness of x and t we write the union as sum $s^* = t+x$. Similarly, $s = t+y$.

Clearly s* and y have no units in common, since the units common to s* and s are all in t. Thus $s^* \cap y = \emptyset$. Hence the sort value of every unit in y is smaller than the sort value of any unit in s* (recall that s* is an initial section of shopping list). Thus, if γ^* is the smallest sort value of elements in s* and $\bar{\gamma}$ is the largest sort value of elements in y, then still $\gamma^* > \bar{\gamma}$

Now the proof is simple. First, if y is empty, s* includes s properly and hence $F(s) < F(s^*)$

If y is not empty, then

$$F(s) = \sum_{(i,v)\in s} c_i \, \gamma_{i,v} = \sum_{(i,v)\in t} c_i \, \gamma_{i,v} + \sum_{(i,v)\in y} c_i \, \gamma_{i,v}$$

$$\leq \sum_{(i,v)\in t} c_i \gamma_{i,v} + \bar{\gamma} C(y)$$

$$< \sum_{(i,v)\in t} c_i \, \gamma_{i,v} + \gamma^* C(y)$$

$$\leq \sum_{(i,v)\in t} c_i \, \gamma_{i,v} + \gamma^* C(x) \leq F(s^*)$$

Hence $F(s) < F(s^*)$, q.e.d.

4. The case (A2)

4.1. Basic Approach

In absence of assumption (A1), two distinct units may have the same sort value. Such units may be of different components. They may also be of the same component, since the assumption (A2) alone does not imply that $\gamma_i(n)$ is strictly decreasing; it merely implies that $\gamma_i(n)$ is monotonically decreasing.

The structure on the shopping list is no longer a total order. It is not even a proper order, since two distinct units with the same sort value will simultaneously precede each other both ways. It is merely a quasi order. However, like any quasi order, it gives rise to a proper order: not on the set of units, but on the set of equivalence classes of units.

To construct this order, the set of units is partitioned into equivalence classes by defining two units as equivalent when their sort values are equal. Thus, for two units, b and c

(4.1.0) Definition: $b \sim c$ if and only if $\gamma(b) = \gamma(c)$

where $b \sim c$ reads: "b is equivalent to c," or: "b and c belong to the same (equivalence) class"

The class to which the unit b belongs is denoted by $[b]$. Thus

(4.1.1) $$[b] = \{X \mid \gamma(X) = \gamma(b)\}$$

that is: $[b]$ is the set of all units X that have the same sort value as b.

The result is that each given sort value determines an equivalence class (namely the class consisting of all units having that given sort value). The set of all equivalence classes is properly ordered; it is, in fact, totally ordered. But within a given class no order is defined (or alternatively, only a quasi order is defined; each unit in the class precedes all units in the class).

Note that some of the classes may contain only one unit, some may contain several units of just one component, some may contain a mixture of units of several components.

In terms of the shopping file this class structure can be visualized by imagining every set of cards with the same sort value as sitting in an envelope--just to indicate that within each envelope there is not meant to be any order.

It is important to note, however, that the class structure requires a substantial change in the shopping file (and in the shopping list). Since within a class there is no order, it is not possible to enter cumulative cost on each card. Therefore cumulative cost is marked only on each envelope, representing the total cost of all classes preceding and including the class in the given envelope.

For the component file that change can be handled by (1) copying the amount from each envelope in the shopping file onto all those cards in the component file that are the duplicates of cards in the envelope; (2) using the following buying rule:

(4.1.2) For each component in the component file, locate the first card on which the cumulative cost entry exceeds the given budget figure, and buy all preceding units.

In sum then, we again have a totally ordered shopping file, provided that it is viewed--not as a file of units, but--as a file of classes. Again:

(4.1.3) Every initial section of that file (of classes) is optimal for its cost.

The proof is abstractly the same as in the case (A1,2). To obtain it explicitly for the present case, all that needs to be done is to replace, in the proof for (A1,2), the term "unit" by the term (equivalence) "class" (and consequently, "cost of the unit" and "sort value for the unit" by "cost of the class" and "sort value for the class"). The procurement is performed by "buying class after class down the list, until the money left is less than the price of the next class on the list."

4.2. Refinement

The basic approach discussed in the preceding subsection is conceptually appealing (in that it reduces the case (A2) to the case (A1,2) by simply using classes instead of units). However, it also causes a substantial loss of flexibility in the procurement function; procuring a class at a time may leave the buyer with a substantial unspendable amount when the next class on the list contains a large number of units.

We would have to leave it at that and consider the (A2) problem as solved, if the loss of flexibility were inherent in the problem. But this is not the case. It is inherent merely in the structure chosen to represent the problem. Hence, in relation to the problem the restriction is artificial, and, as such, removable.

To that end all that is needed is to provide, for the last step in each procurement process, the option of buying part of the next class. To provide that option requires some additional structure in the shopping list (or file) in order to make it compatible with the procurement process it was intended to represent.

Recall that within an equivalence class no (proper) order has been defined, and assume the buying is done directly from the shelves in the store. The buyer is just ready to perform the last step. The next class L contains, among other things, several units of Component D. The classes he has bought contain a total of 6 units of Component D. If he now pulls an arbitrary unit of D from the shelf, it will become his 7th unit of that component. If he buys another one, it will be his 8th. Thus, in the envisioned procurement process, buying the units one by one generates a proper order (which can then be expressed by ordinal numbers: the 7th, the 8th) even when their associated sort values are equal. Therefore, in the shopping list, where all the units of a given component have been numbered in advance, this numbering must be used as an order (even when sort values are equal).

This is achieved by introducing within each class a so-called partial order, relating only units of the same component while leaving units of different components unrelated.

Explicitly, this partial order within each class together with the total order among classes is defined for the case (A2), by

(4.2.1) Definition: (i,m) precedes (j ,n) if and only if

either (I) $\gamma_{i,m} > \gamma_{j,n}$

or (II) $\gamma_{i,m} = \gamma_{j,n}$ and $i = j$ and $m < n$

where (I) describes the total order among classes and (II) defines the partial order within a class; note that for each fixed component its units are totally ordered.

(4.2.2) Definition: An initial semi-section of the shopping list is the union of an initial section of the totally ordered set of classes and a part of the next class, provided that the chosen part contains, with any unit of a given component, also all the preceding units of the same component in that next class.

Obviously, the restriction on the nature of a part, as defined above, is merely a formal one. On the shelves any part of the next class will qualify; on the shopping list the requirement is that among units of a given component the lowest units in the class should be chosen.

(4.2.3) An initial semi-section is optimal for its cost.

The proof is left to the reader. Note that uniqueness of solution no longer holds (any two parts of the "next class" having the same total costs can be interchanged).

5. The General Case

For the problem at hand the case discussed in the preceding section is not a very special one. In a macro or global sense the assumption (A2) will hold. That is, for each fixed i, the general trend of the function $\gamma_i(n)$ will be to monotonically decrease. Deviations from this property will be of a merely

local nature. But whenever these deviations occur assumption (A2) does not hold, and this creates significant difficulties in solving the problem--no matter which approach is taken.

In the continuous case the classical approach is the LaGrange multiplier method which solves for a given funding constraint. A necessary condition for the solution to be a maximum is negative second order partial derivatives--the continuous form of (A2). For the discrete case (the problem at hand) (A2) itself is a necessary condition. In the proposed approach with the LMI Shopping List Model, which solves without regard to funding constraints (solves for all funding levels simultaneously), (A2) is a necessary condition for the very construction of the shopping list. (A2) merely states that, for every fixed component the natural order of its units must be compatible with the shopping list order induced by the sort values.

As an example, footnote 1 on page 15 describes a hypothetical but realistic situation involving airplane brakes which fail infrequently but usually fail in pairs. In such situations one spare brake might be of little value ($\gamma_{B,1}$ might be small) while two spare brakes might provide a large NORS reduction ($\gamma_{B,2}$ might be large). Taken at face value the second unit in this example would provide a greater improvement than the first. Thus unless adjustments are made the LMI NORS Model would generate a procurement plan indicating that the second unit should be procured before the first--a logical absurdity.

In the simple brake example the solution is obvious. The first and second spare brakes should be procured as a pair. Each of the two spare brakes would then provide the average worth of the two.

In a more general case an averaging technique is required to assure that the sort value of the n^{th} spare of a given component is not greater than that of the $n-1^{th}$ spare.

Mathematically the averaging effect is to smooth out local reversals in monotoneity so as to make each γ_i a monotonically decreasing function of n. This, of course, changes the original problem P to a new problem $\overline{P}$, which has the property that every solution of P is also a solution of $\overline{P}$ whereas not every solution of $\overline{P}$ is a solution of P. Thus it will also be necessary to insure that only those solutions of $\overline{P}$ are used that are also solutions of P.

5.1. The averaging procedure

Assume for some fixed i, the sequence of sort values

$\gamma_i(1),\ \gamma_i(2),\ \gamma_i(3),\ \ldots\ \gamma_i(K)$

is not monotonically decreasing. Say this sequence is

.9, .7, .7, .8, .6, ...

The first three terms are monotonically decreasing, so are the last two. But the third and fourth are not. If both are replaced by their average, .75, those two terms will be monotonically decreasing.

However, the new sequence then reads

.9, .7, .75, .75, .6 ...

and now the second and third terms are not monotonically decreasing. We could replace these two by their average, obtaining another new sequence

.9, .725, .725, .75, .6 ...

again not monotonically decreasing. We could continue in this manner. However, it is desirable to have a systematic procedure for achieving the objective of a monotonically decreasing sequence.

For this purpose a few definitions are needed. Let A be a sequence of real numbers

$$A = (a_1, a_2, \ldots a_n, \ldots a_K)$$

A set of consecutive terms in the sequence will be called a chain. If that chain is monotonically increasing it will be called an MIC (monotonically increasing chain).

An XMIC is a MIC that is maximal in the sequence; that is, if the preceding term is adjoined to it on the left or the succeeding term is adjoined to it on the right, then it is no longer an MIC.

For example, in any sequence, a single term is always an MIC; in the sequence (9, 7, 7, 8, 6) there are exactly three XMIC's namely

(9), (7,7,8), (6).

Note that for any sequence A it is true that the set of XMIC's partitions A (proof is trivial).

Now the procedure to be applied, for each fixed component, to the sequence of sort values of its units is described as follows.

Step 1. Given a sequence, if it is monotonically decreasing, it is finished. Otherwise go to Step 2.

Step 2. Partition the sequence into its XMIC's, and, in each XMIC, replace each term by the average of all terms. Consider the new sequence so obtained as the given sequence and go to Step 1.

The procedure has the following properties.

If the originally given sequence is finite, as is the case in the problem at hand, the procedure will terminate (i.e., will yield a monotonically decreasing sequence) after a finite number of steps.

If the original sequence is denoted by A^o, the result of one iteration by A^1, of two iterations by A^2, etc., then:

An XMIC in A^{n+1} is the union of (consecutive) XMIC's in A^n, and the average of an XMIC in A^{n+1} is the average of the corresponding union of XMIC's in A^n. By induction this implies that, for any k, an XMIC in A^k is the union of XMIC's in A^o and the average of an XMIC in A^k is the average of the corresponding union of XMIC's in A^o.

Thus, in particular, if k is the final iteration, the sequence A^k could have been obtained directly from A^o by averaging certain unions of XMIC's in A^o. So why then did we apply that iterative procedure? Because we did not know which unions.

5.2. Characterization of solutions

The averaging procedure associates with each sequence of sort values

$$\gamma_i(1),\ \gamma_i(2),\ \ldots,\ \gamma_i(K)$$

a new sequence, say

$$\rho_i(1),\ \rho_i(2),\ \ldots,\ \rho_i(K)$$

which is monotonically decreasing, and hence satisfies the assumption (A2).

Thus, if, instead of the original sort values $\gamma_{i,n}$, the new values $\rho_{i,n}$ are used to construct the shopping list, the new

problem will be the case (A2) discussed in Section 4; the shopping list will then satisfy (4.2.3), so that every initial semi-section is a solution.

However, as remarked earlier, not every solution of the new problem (call it the problem $\overline{P}$) is a solution of the original problem P. The situation is roughly as follows: A solution of $\overline{P}$ is a solution of P when it occurs at a "location" (heuristically: a stopping point in the shopping list; mathematically: a vector in the vectorspace of stocklevels) where no averaging has taken place.

Explicitly, the following simple statement holds.

(5.2.1) Let s* be a solution of $\overline{P}$, that is let s* be an initial semi-section of the shopping file of classes ordered according to the sort values $\rho_{i,n}$; let C denote the part of the last class used in the semi-section, and D the part omitted. If every unit (i, n) in D satisfies

$$\rho_{i,n} = \gamma_{i,n}$$

then s* is a solution of P (hence optimal for its cost).

The proof is left to the reader, it is again methodologically the same as for statement (3.1.1), except for the necessary further details, which are straightforward.

6. The NORS-NFE Model

For this description it is assumed that two types of NFE are distinguished: Serious and Minor ones. It is further assumed that, for a given A/C type, the losses in operating effectiveness caused by these NFE's are judged to compare as follows:

$$1 \text{ Serious NFE} \sim \alpha \times 1 \text{ NORS}$$

$$1 \text{ Minor NFE} \sim \beta \times 1 \text{ NORS}$$

where $$0 < \beta < \alpha < 1$$

The overall NORS-NFE problem is then concerned with procurement of spares for three kinds of components, those that cause NORS, serious NFE and minor NFE.

It is relevant to observe: if we were to view the overall problem as consisting of three separate independent problems these would be abstractly identical. Hence, in that case we would simply construct three shopping lists - one for each problem.

Obviously, the three problems together are not equivalent to the overall problem, since giving a budget for the overall procurement is not equivalent to giving three separate budgets, one for each class of components. Hence, as parts of the overall problem, the three problems are not independent.

However, it will be shown that the overall problem can be solved in three steps: the first consists precisely in constructing the three shopping lists (Sec. 6.1), the second relates

these lists to the overall problem (Sec. 6.2) while the third describes the procedure for merging them into a single shopping list for the overall problem (Sec. 6.3).

6.1. The NORS and the NFE Shopping List

Notation. For NORS causing components the notation

$$s;\ s_i;\ p_i(s_i);\ q_i(s_i);\ \ q_i(n);\ q_{i,n};\ \gamma_{i,n};\ \gamma_i(n)$$

as defined in (1.1) and (1.3) is retained. For components causing serious NFE's and for those causing minor NFE's the corresponding entities will be denoted by

$$s';\ s'_j;\ p'_j(s'_j);\ q'_j(s'_j);\ q'_j(m);\ q'_{j,m};\gamma'_{j,m};\ \gamma'_j(m)$$

and $s'';\ s''_v;\ p''_v(s''_v);\ q''_v(s''_v);\ q''_v(k);\ q''_{v,k};\ \gamma''_{v,k};\ \gamma''_v(k)$

respectively, and assume the probabilities, q, q', q'' are all mutually independent.

Further, denote by

$Q(s) = Q_s = \pi q_i(s_i);\ Q'(s') = Q'_{s'} = \pi q'_i(s'_i);\ Q''(s'') = Q''_{s''} = \pi q''_i(s''_i)$

$z = s + s' + s''$ = overall stock level.

Viewing each of the three problems separately, the $\gamma_{i,n}$ determine the shopping list for the NORS causing components, while the $\gamma'_{j,m}$ and the $\gamma''_{v,k}$ determine the two shopping lists for the major NFE and the minor NFE causing components, respectively.

Each of the three shopping lists solves its associated problem and has all the properties of the shopping list discussed in Sec. 2-5. For instance, the last of the three lists solves the problem:

> determine all optimal stock levels for minor NFE causing components

in the sense that the list satisfies (3.1.1), (4.1.3) and (4.2.3).

6.2. The NORS-NFE Problem

To combine the three lists in a manner that solves the overall problem, all that is needed in this case is knowledge of the function to be maximized.

In the NORS problem that function was $Q(s)$, or equivalently, $N \cdot Q(s)$ = the expected number of not-NORS aircraft (assuming just one type of aircraft).

Here in the overall problem, however, three classes of not-NORS aircraft must be distinguished, namely

FE(fully equipped); Minor NFE; Serious NFE.

For a given overall stock level z, the expected number of aircraft in each of these classes is

$$N \cdot Q(s) \cdot Q'(s') \cdot Q''(s''); \qquad N \cdot Q(s) \cdot Q'(s') \cdot \left[1 - Q''(s'')\right]; \qquad N \cdot Q(s) \cdot \left[1 - Q'(s')\right]$$

or, omitting the arguments

$$N\,Q\,Q'\,Q'' \qquad N\,Q\,Q'\left[1-Q''\right] \qquad N\,Q\left[1-Q'\right]$$

The values of an A/C in these classes relate as

$$1 \qquad (1-\beta) \qquad (1-\alpha)$$

or, setting $\bar{\beta} = 1-\beta$, $\bar{\alpha} = 1-\alpha$, the A/C values relate as

$$1 \qquad \bar{\beta} \qquad \bar{\alpha}$$

The problem is to maximize the total value of the three classes,

$$
\begin{aligned}
\text{which} &= N[Q\,Q'Q'' + \bar{\beta}\,Q\,Q'(1-Q'') + \bar{\alpha}\,Q(1-Q')] \\
&= N\,Q[Q'Q'' + \bar{\beta}\,Q'(1-Q'') + \bar{\alpha}(1-Q')] \\
&= N\,Q[Q'Q''(1-\bar{\beta}) + Q'(\bar{\beta}-\bar{\alpha}) + \bar{\alpha}] \\
&= N\,Q[Q'(\beta Q'' + \alpha - \beta) + \bar{\alpha}] \\
&= N\,G
\end{aligned}
$$

where

$$G = Q[Q'(\beta Q'' + \alpha - \beta) + \bar{\alpha}] \text{, or more explicitly:}$$

$$(6.2.1) \qquad G(z) = Q(s)\left[Q'(s')\left(\beta Q''(s'') + \alpha - \beta\right) + \bar{\alpha}\right]$$

Thus the problem here is to maximize NG. This is equivalent to maximizing G (since N, the number of aircraft, was assumed to be constant). G is a function of z; z is the overall stock level for the three categories of components, that is $z = s + s' + s''$. (6.2.1) defines $G(z)$ in terms of the three functions $Q(s)$, $Q'(s')$ and $Q''(s'')$.

Some properties of the function $G(z)$ will be needed. For a given stock level $z = s + s' + s''$, denote by A, A' and A" the next unit of some component in the NORS, serious NFE, and minor

NFE causing category, respectively. When one of these next units is added to the given stock level z, the effect--per unit of its cost--will be to multiply in (6.2.1)

$$Q \text{ by } \lambda(A), \quad \text{or } Q' \text{ by } \lambda'(A'), \text{ or } Q'' \text{ by } \lambda''(A'')$$

as is readily seen from (1.3.7).

Denote by C, C', C" the cost of A, A', A" and use the same symbolic notation as in (1.3.7), that is write $\frac{A}{C}$, etc., to express the "per unit of cost" specifications. Then the effect of an added unit, per unit of its cost, is simply described as increasing G(z) to $G\left(z + \frac{A}{C}\right)$. Omitting all arguments, the value of the latter expression is then given by

$$(6.2.2) \quad \begin{cases} G\left(z + \frac{A}{C}\right) = \lambda Q\left[Q'(\beta Q'' + \alpha - \beta) + \bar{\alpha}\right] \\ G\left(z + \frac{A'}{C'}\right) = Q\left[\lambda' Q'(\beta Q'' + \alpha - \beta) + \bar{\alpha}\right] \\ G\left(z + \frac{A''}{C''}\right) = Q\left[Q'(\beta \lambda'' Q'' + \alpha - \beta) + \bar{\alpha}\right] \end{cases}$$

where $\quad z = s + s' + s'', \; Q = Q(s), \; Q' = Q'(s'), \; Q'' = Q''(s'')$

Obviously, the three expressions in (6.2.2) play the role of sort values (of A, A', A") for the overall problem.

The original sort values λ, λ', λ" had the outstanding property of depending only on the individual unit of the given component, and hence they could be computed before assembling the shopping list. The new sort values for the overall problem, or briefly, the <u>overall sort values</u>, do not always have that property, and hence must be computed--from (6.2.2)--while assembling the overall shopping list. Specifically, the situation is as follows.

<u>Within</u> any one of the three categories the new overall sort values have the same order as the original ones, that is

$$
(6.2.3)\quad \begin{cases} G\left(z + \frac{A}{C}\right) > G\left(z + \frac{\overline{A}}{\overline{C}}\right) \Longleftrightarrow \lambda > \overline{\lambda} \\ G\left(z + \frac{A'}{C'}\right) > G\left(z + \frac{\overline{A'}}{\overline{C'}}\right) \Longleftrightarrow \lambda > \overline{\lambda'} \\ G\left(z + \frac{A''}{C''}\right) > G\left(z + \frac{\overline{A''}}{\overline{C''}}\right) \Longleftrightarrow \lambda > \overline{\lambda''} \end{cases}
$$

The proof is immediate from (6.2.2) and the observation that

$$Q,\ Q',\ Q'',\ \alpha,\ \beta,\ \alpha-\beta,\ \overline{\alpha}$$

are all positive.

For units from two <u>different</u> categories, the overall sort values may not be in the same order relation as the original sort values.

The effect of the two outlined circumstances on the structure of the overall shopping list is as follows.

The overall list <u>preserves the order</u> between units from the <u>same</u> original list and <u>establishes an order</u> between units from <u>different</u> original lists. Thus the problem of constructing the overall shopping list is not a problem of establishing an (entirely new) order among the set of all units from the 3 original lists, but merely a problem of <u>merging</u> the 3 lists (without disturbing the order in any of them).

After these preparations, the following description of the shopping list for the overall problem will be self-explanatory.

6.3 The NORS-NFE Shopping List

To obtain the shopping list for the overall NORS-NFE problem, assume that for each of the three component categories a shopping list has been constructed by treating the procurement

for that category as if it were an independent problem, without relation to the other two categories. Assume further that those constructions have been made for the general case, thus including application of the averaging procedure, wherever necessary, for leading to the new sort values ρ, ρ', ρ''.

The three shopping lists will briefly be referred to as the

NORS, NFE, and nfe

list where NFE and nfe stand for the serious-NFE and the minor-NFE category, respectively.

Finally, for computational convenience, assume that each list consists of five columns, so that each row carries five entries. For the NORS list these entries are denoted by

(i,n) = component-unit identification

ρ = the sort value $\rho_{i,n}$

ω = e^{ρ} (so that $\rho = \mathrm{Log}\ \omega$)

Q = Q(s) for the stock level s reached with this entry on the list

C = the unit cost

For the nfe list and the NFE list they are denoted by

$(i,n)'$, ρ', ω', Q', C' and $(i,n)''$, ρ'', ω'', Q'', C''

After these preparations, assembly of the three lists into a single shopping list for the overall problem is achieved by the "merging procedure" described below.

Denote by L the list to be constructed. It will consist of 3 columns, thus each row in L will have 3 entries.

In order to loosen up somewhat the stiffness in that formal description, it is being preceded here by an informal, imprecise, but more suggestive outline, and will also be followed by explanatory notes.

Informal Outline. Units from the three original lists are moved, one by one, to the new combined list L. On each original list the units are chosen in the order listed. Thus, the first 3 candidates for a move to L are the three units topping the original lists. A decision must be made which of the three to admit first. For this purpose they are led into a clearing house S. At the desk they find 3 badges designating their origins: A, A', A''. Each one takes the appropriate badge and puts his passport in its place on the desk. Then they are seated in chairs marked with the same designations.

Only at this point does the procedure begin to be iterative.

The official at the desk bases his decision on information from the passports. To this information he applies a formula to obtain a set T of 3 numerical values: one for A, one for A', one for A''. Then he applies the rule: "the unit with the largest value wins admission to L" and announces the winner. If he announces, say A', then A' steps up to the desk, takes his passport, putting the badge in its place, and goes on to L.

In the clearing house, the official's announcement directed A' to exchange documents and leave for L; however, at the headquarters of the original shopping lists the same announcement signifies a call for the next unit. Thus, when A' leaves S, the new unit - from the same origin - enters S,

exchanges his passport for the only badge on the desk, and is seated in the vacated chair. The official at the desk has again 3 passports and proceeds as before.

At L they have a log where each new arrival is recorded. It provides 3 columns:

the name column, for unit identification

the G column, the G-value from (6.2.1) for the new stock level z

the C column, for cumulative cost.

At the beginning the row (or line) #0 has been filled in:

a blank in the name column

the G value from (6.2.1) for $s = s' = s'' = 0$

0 in the C column.

Then, for each new arrival, a row is filled in as follows: the name (e.g., $(17.5)'$ --to mean: 5th unit of component 17 from the category of serious-NFE-causing components) is copied from the passport, the G-value is computed by (6.2.1), while the C-value is obtained by adding the unit's cost (from the passport) to the C-value in the preceding row.

This log is then the new shopping list. The formal description follows:

(6.3.1) Merging Procedure

Step 0. a) In row #0 of L, enter the value of G(0) from (6.2.1) in the G-column.

b) Take the first unit in the NORS list, the NFE list and the nfe list; call them A, A', A''; the set they form call S, thus $S = \{A, A', A''\}$;

c) Compute the three values in the set

$$T = \left\{ G\left(0+\frac{A}{C}\right),\ G\left(0+\frac{A'}{C'}\right),\ G\left(0+\frac{A''}{C''}\right) \right\}$$ by (6.2.2),

after replacing the lambdas by the omegas (page 89).

Step 1. a) The unit $X \in S$, which corresponds to the largest value in T, enter--under its original name--in the unit column of the next row in L.

b) The new stock level just achieved denote by z, compute G(z) by (6.2.1) and enter the value in the G-column of L. Enter C(z) in the C-column of L.

c) In S, replace the unit X by the next unit from its original list, calling this new unit again X (i.e., A, or A', or A'') and the new set again S.

d) Compute the values in the set

$$T = \left\{ G\left(z + \frac{A}{C}\right),\ G\left(z + \frac{A'}{C'}\right),\ G\left(z + \frac{A''}{C''}\right) \right\}$$ by

(6.2.2) after replacing the lambdas by the omegas.

e) Repeat step 1.